Sanity Check with Wavelets

Data

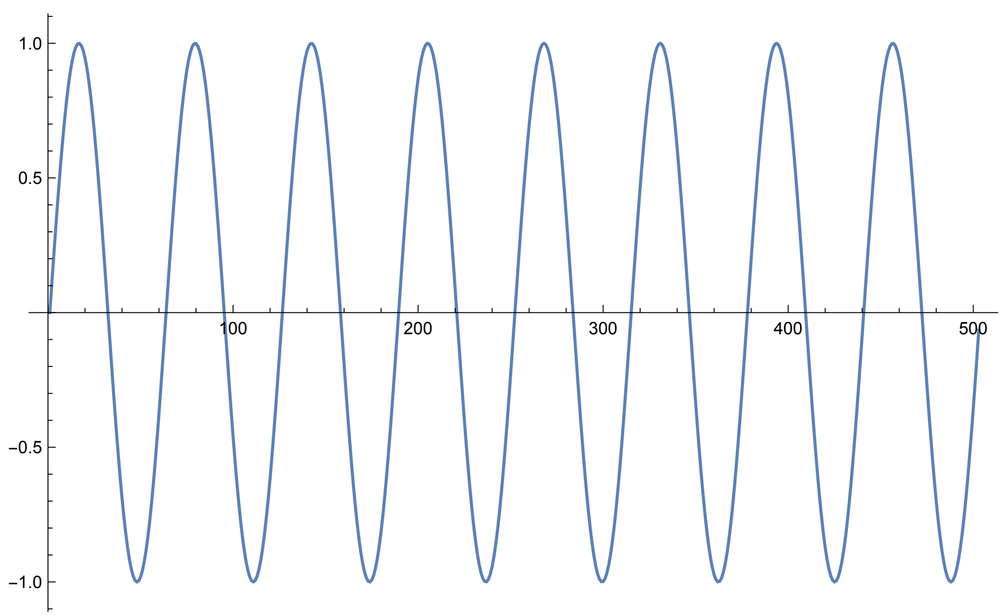
$\mathrm{Sin}(x) = y \quad -8\pi \leqslant x \leqslant 8\pi$ increments of $+0.1$ total of 503 samples equally spaced.

Periodicity of about 76 along x-axis.

Localized Frequency: Frequency for short regions of x
Localized Periodicity: $\frac{1}{\text{Localized Frequency}}$

This concept of Localized Periodicity and Frequency might require serious adjustment of investigations.

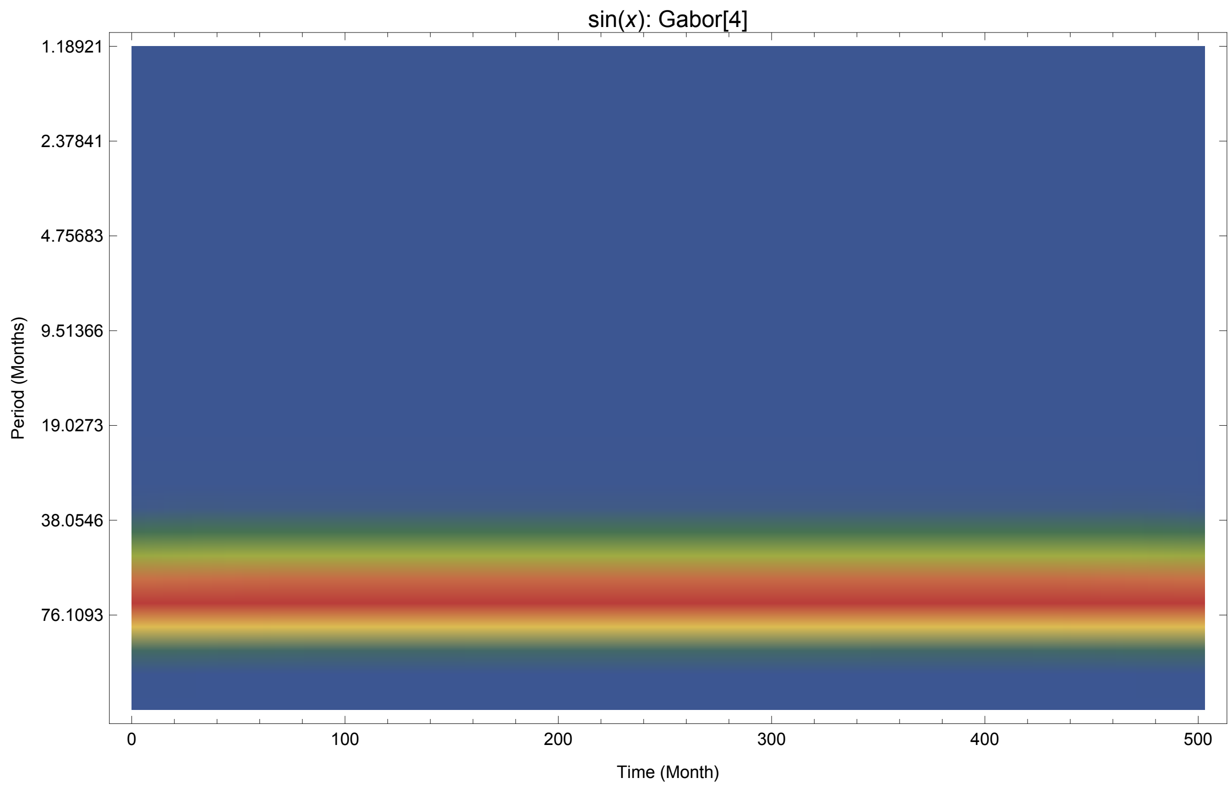
In Scalogram heat-maps below, the y-axis is Periodicity in the same unit of time as the x-axis, in months (for the sake of El-Nino comparison plots).



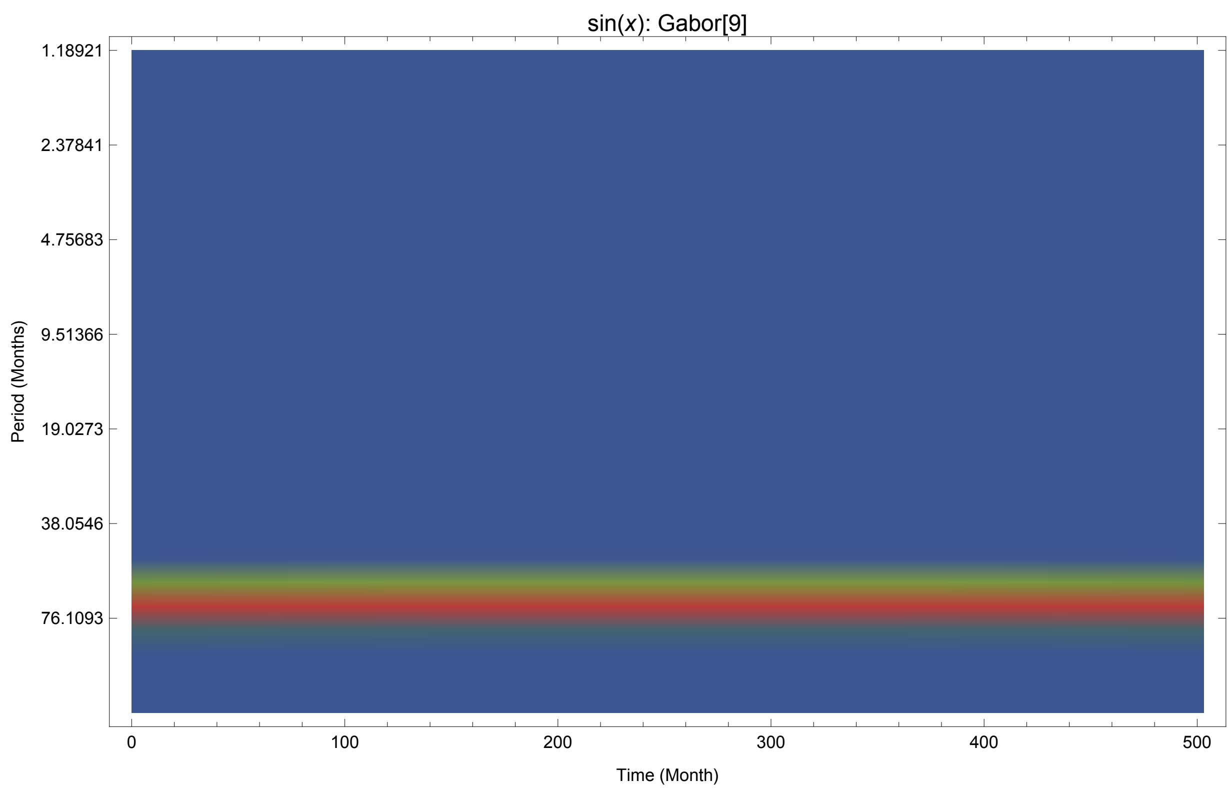
Gabor Wavelet captures the periodicity:

≈ 60

Out[416]=

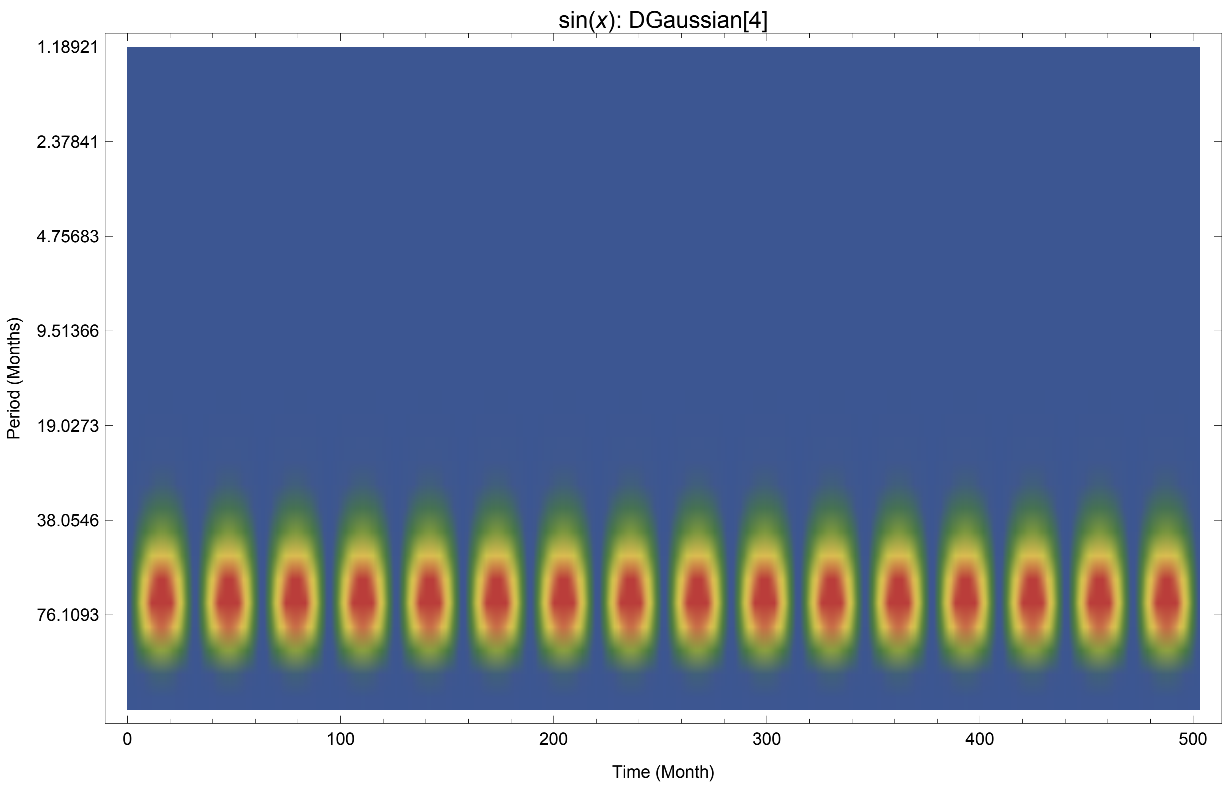


Out[541]=



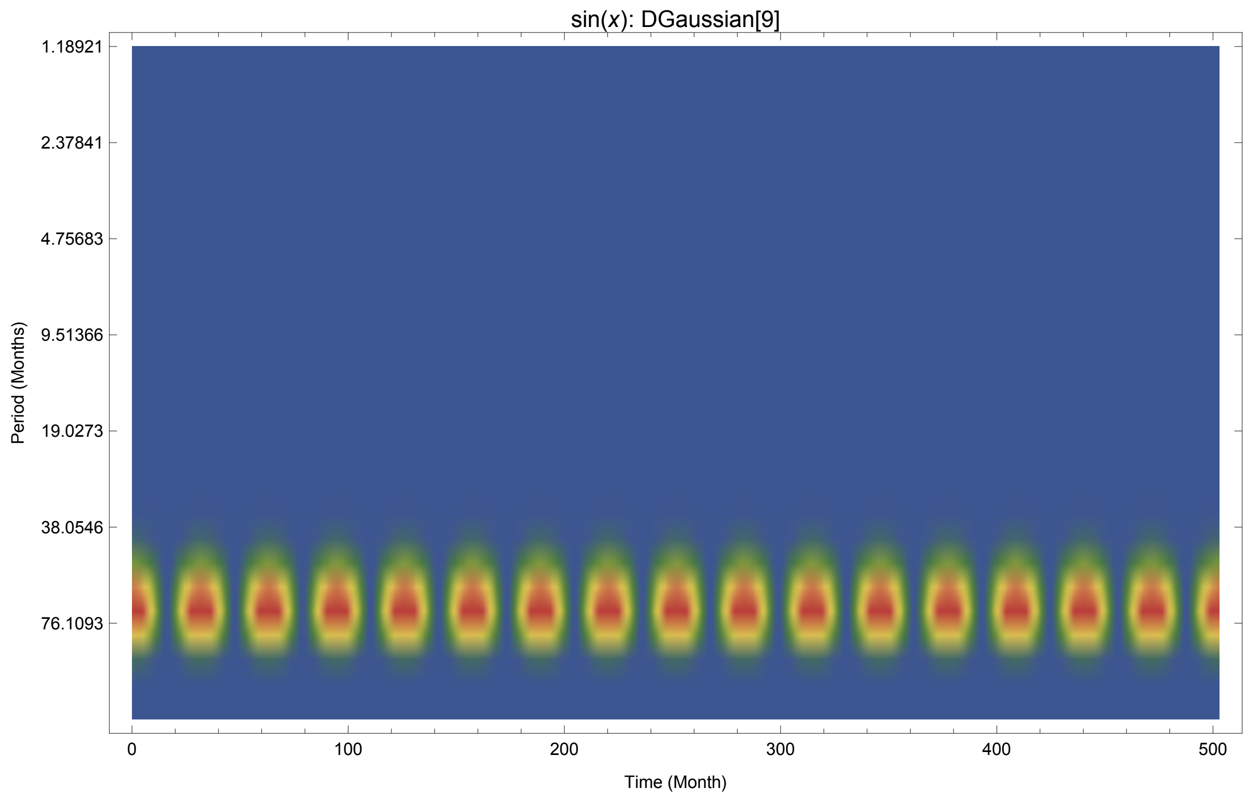
Artefact observed for DGaussian, seemingly the amplitude does drop to Min periodically for the y-value of period:

Out[434]=



Increasing the order of the Gaussian helps a little, not by much.

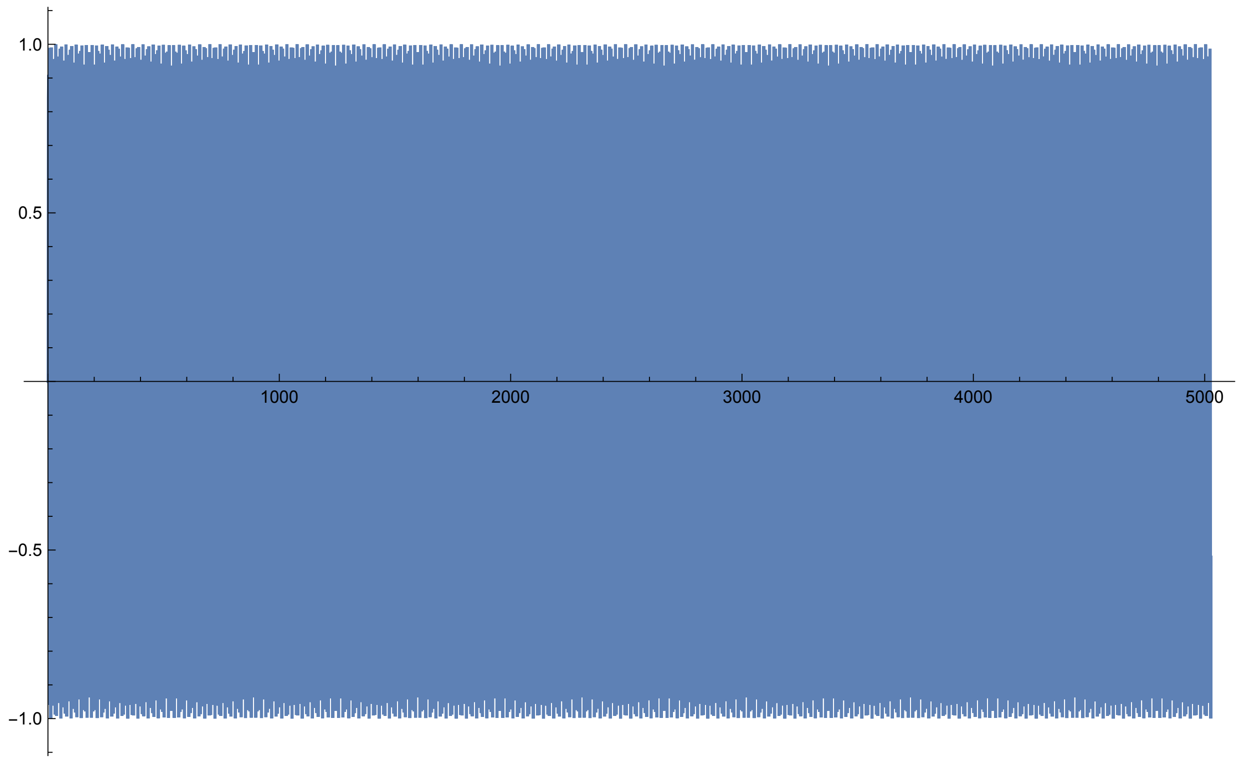
Out[440]=



Let’s investigate the DGaussian artefact by increasing both the frequency of the Sin as well as the sampling rate:

$\text{Sin}(100\,x) = y \quad -8\,\pi \leqslant x \leqslant 8\,\pi$ increments of $+0.01$

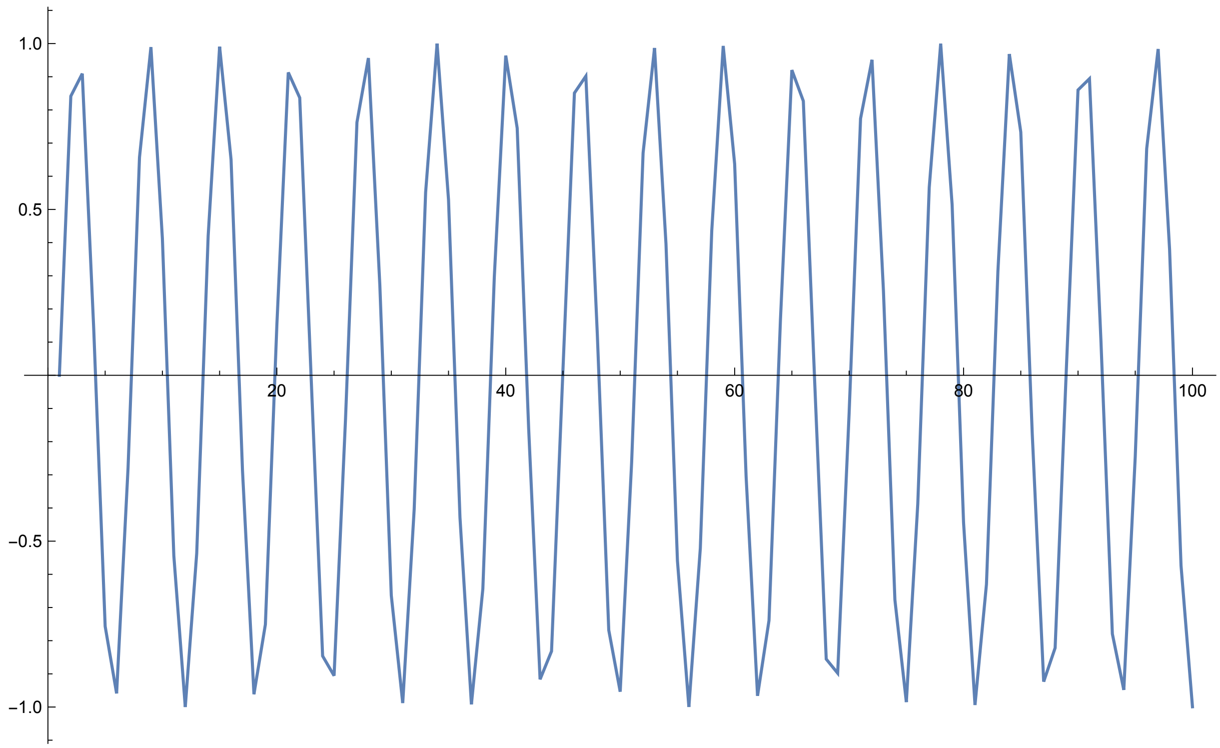
Too periodic to properly plot:



Let’s look at first 100 samples, periodicity if about:

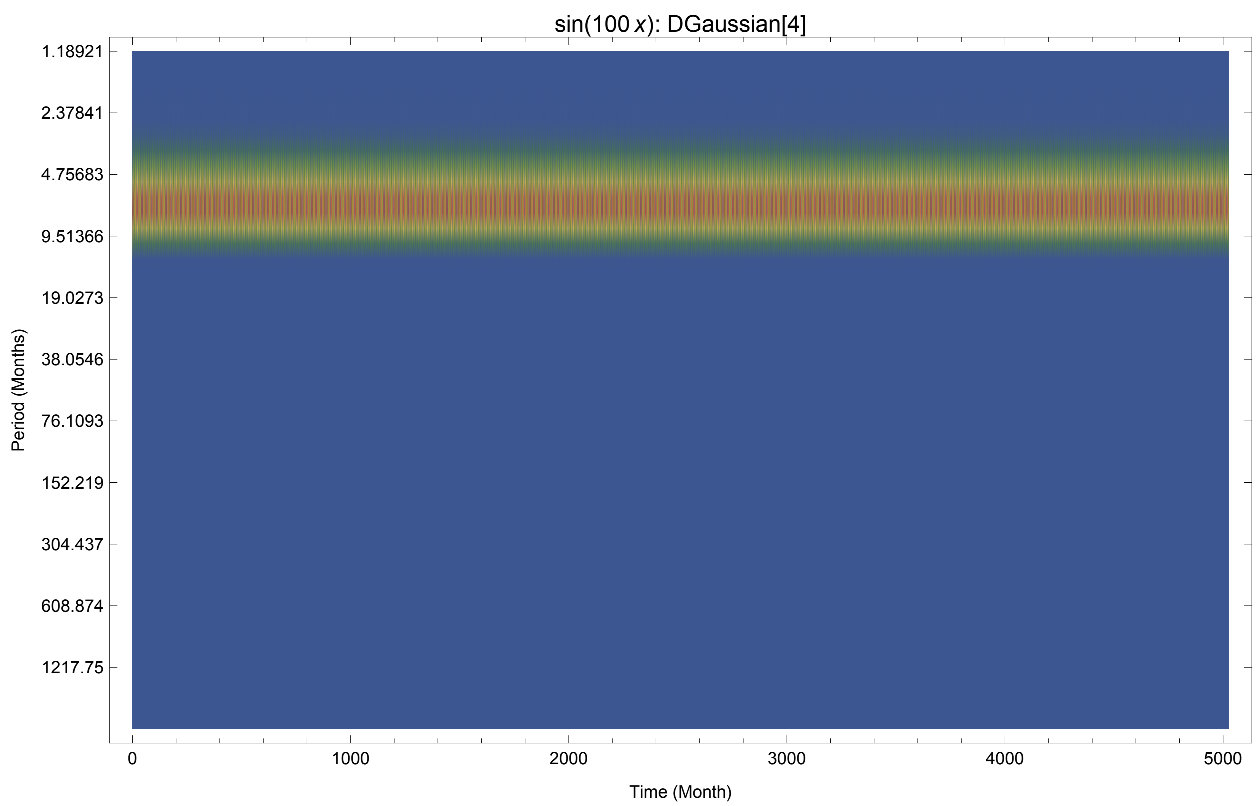
≈ 6-7

Out[606]=



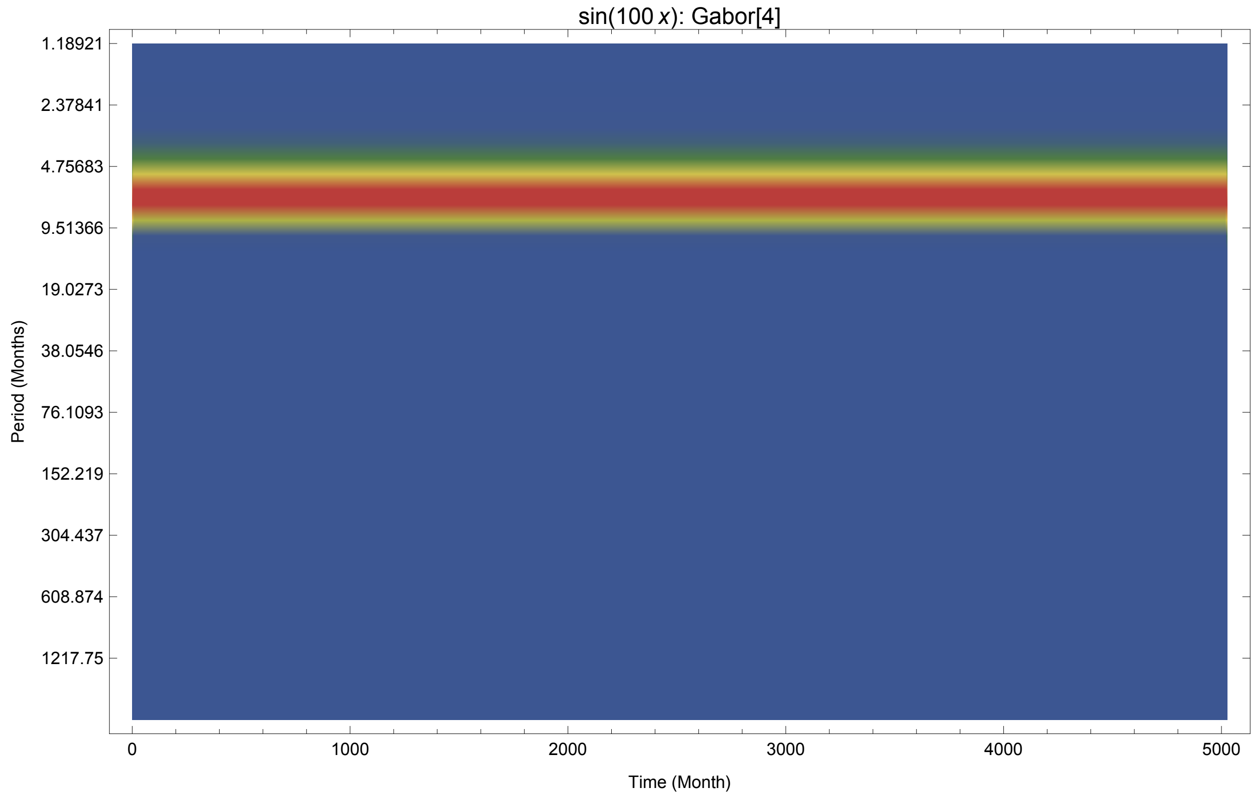
DGaussian of order 4 caputres the periodicity correctly but the above mentioned artefact is gone! Therefore that artefact has to do with the sampling rates.

Out[567]=

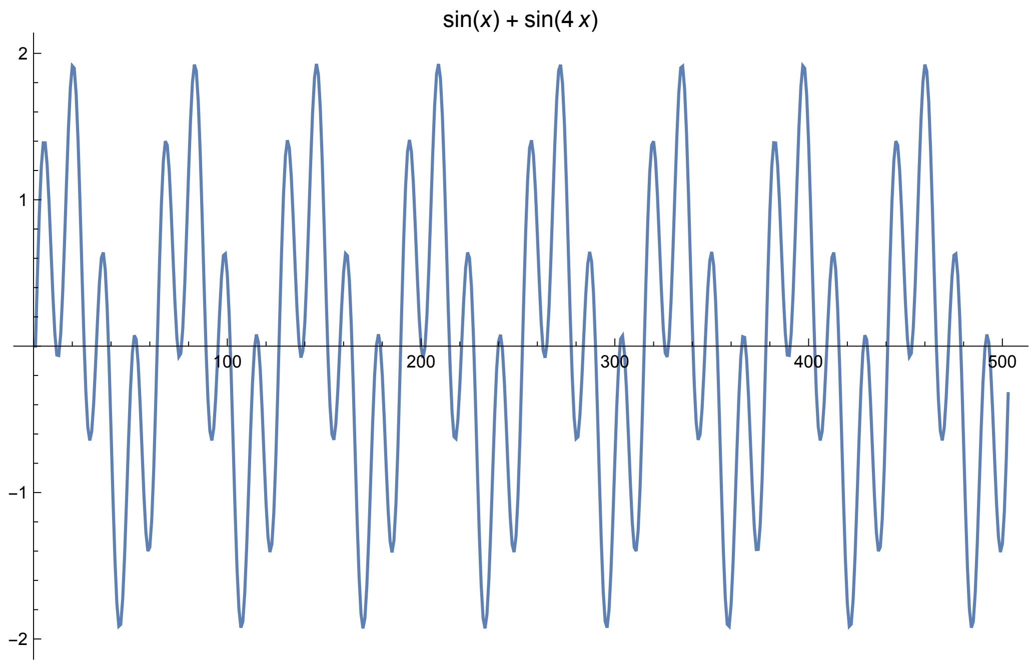


Gabor captures the same periodicity:

Out[573]=

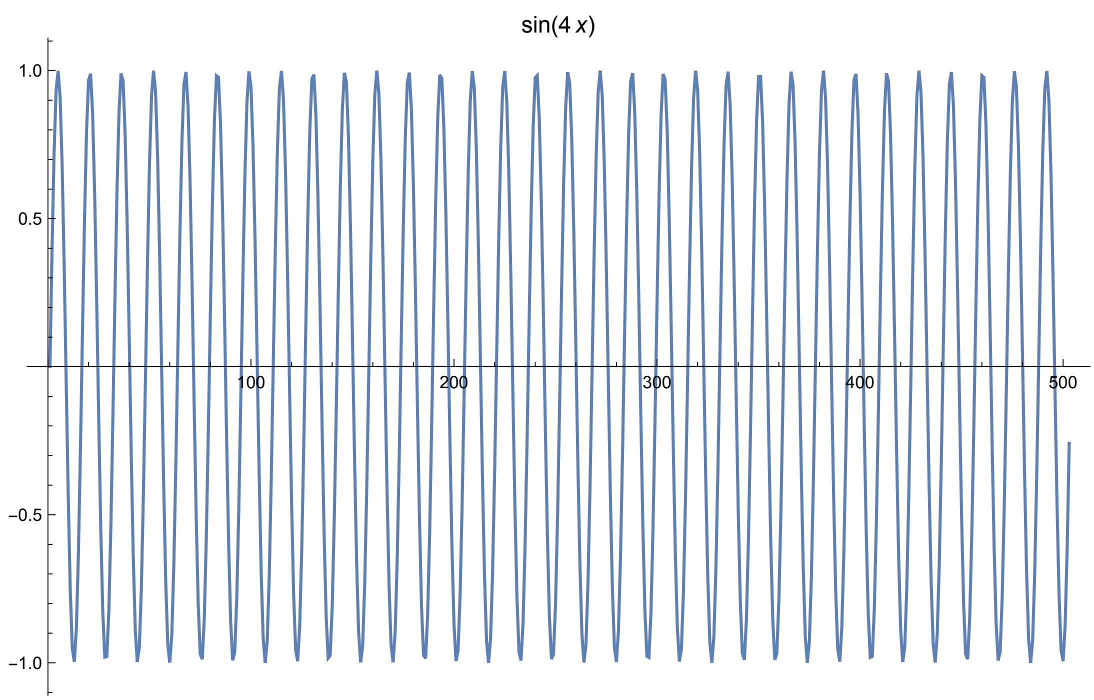


Multiple Trends



Periodicity for Sin(4x) is

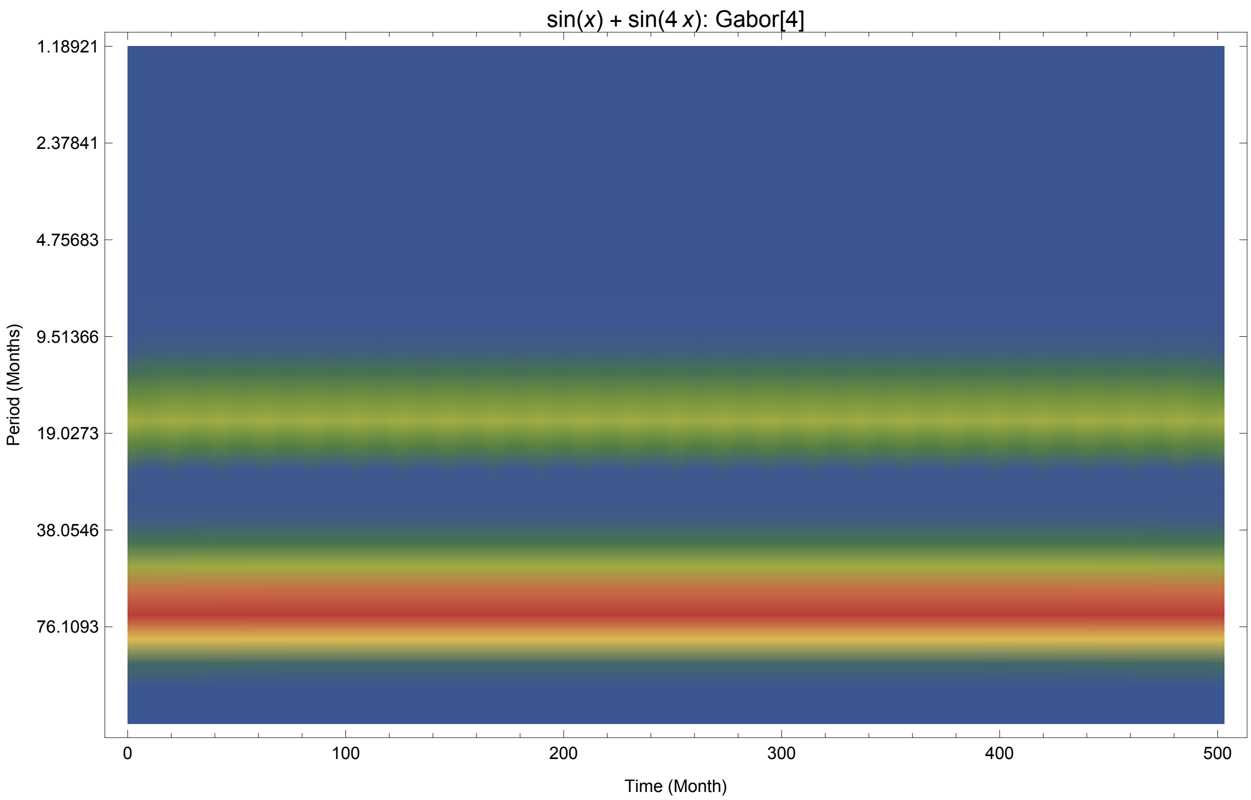
≈ 18-19

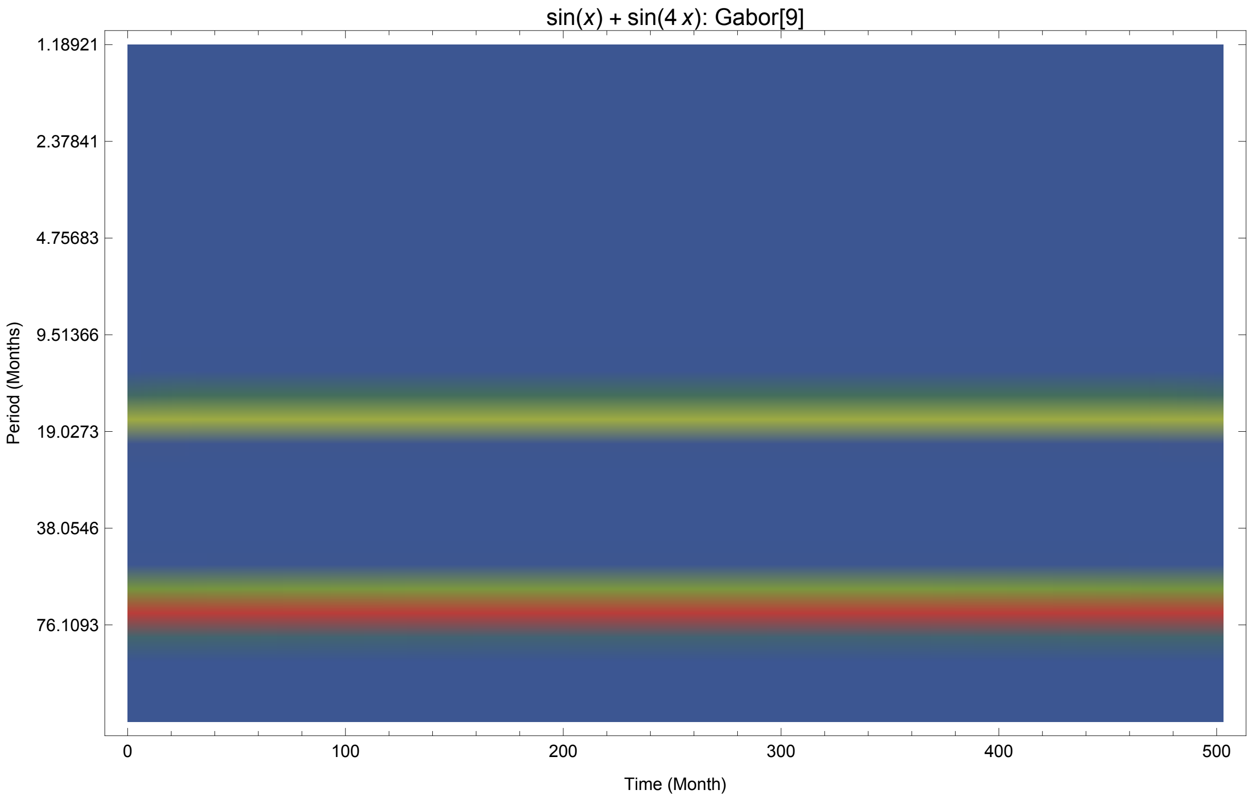


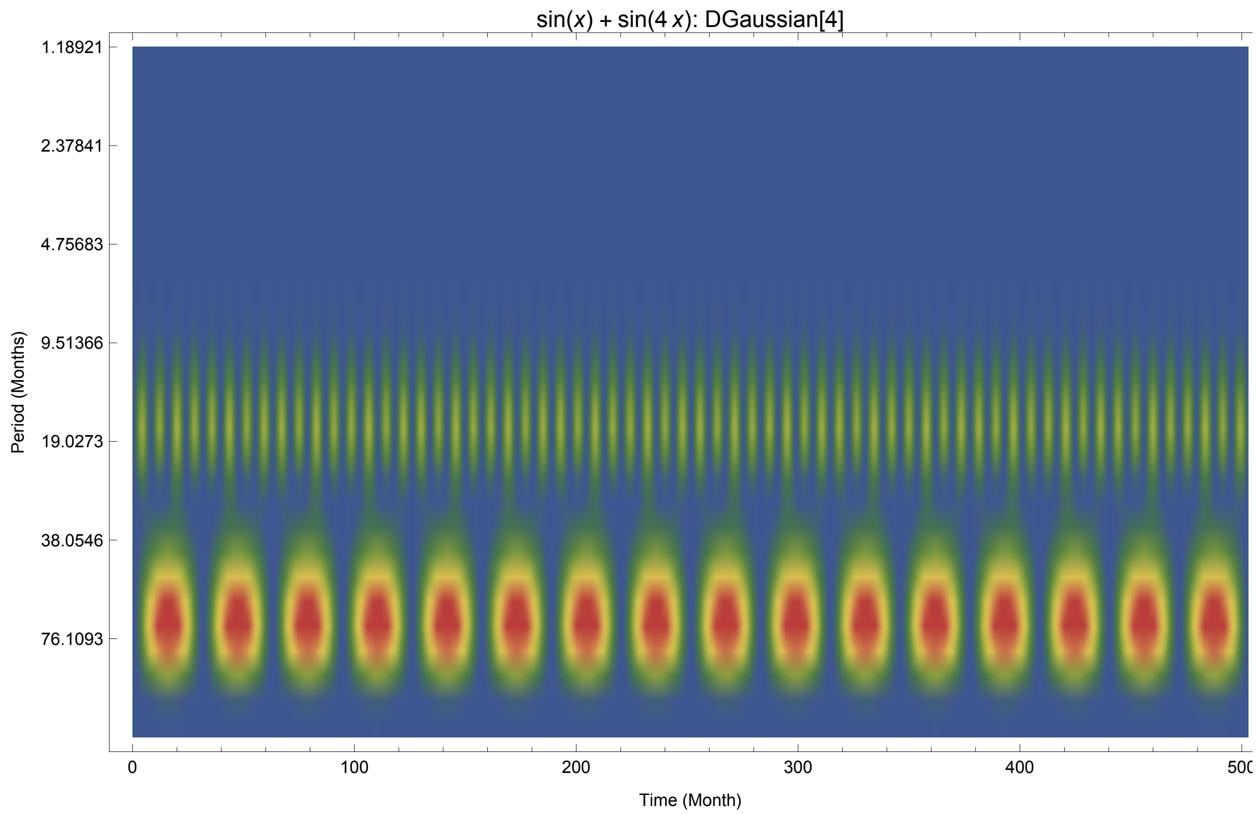
Gabor captures the same periodicity(s):

≈ 18-19

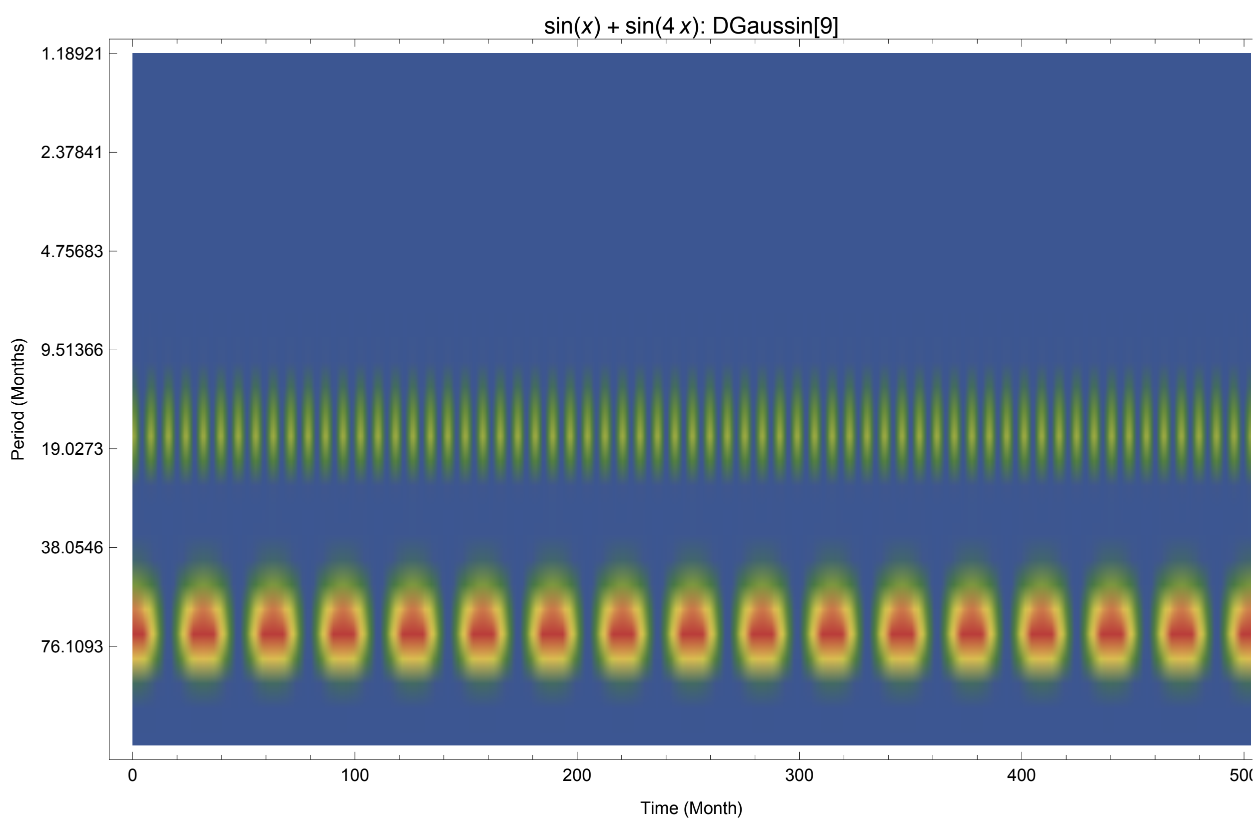
≈ 60


sin(x) + sin(4 x): Gabor[4]
1.18921
2.37841
4.75683
9.51366
19.0273
38.0546
76.1093
Period (Months)
0
100
200
300
400
500
Time (Month)


sin(x) + sin(4 x): Gabor[9]
1.18921
2.37841
4.75683
9.51366
19.0273
38.0546
76.1093
Period (Months)
0
100
200
300
400
500
Time (Month)


sin(x) + sin(4 x): DGaussian[4]
1.18921
2.37841
4.75683
9.51366
19.0273
38.0546
76.1093
Period (Months)
0
100
200
300
400
500
Time (Month)

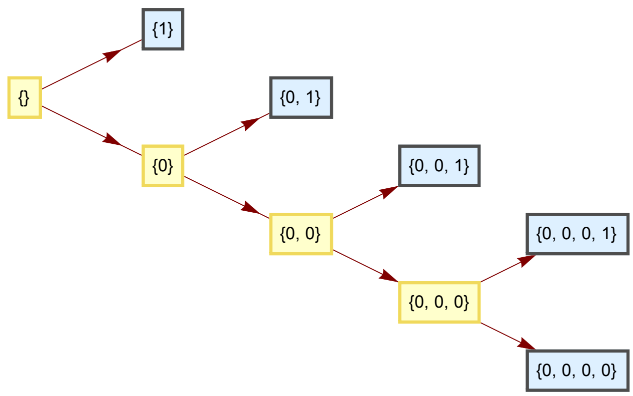
Out[642]=


sin(x) + sin(4 x): DGaussin[9]
1.18921
2.37841
4.75683
9.51366
19.0273
38.0546
76.1093
Period (Months)
0
100
200
300
400
500
Time (Month)

Tahiti Decomposition Scalograms

Tahiti data decomposed using DaubechiesWavelet[4],4,Padding→"Extrapolated"

Out[9]=

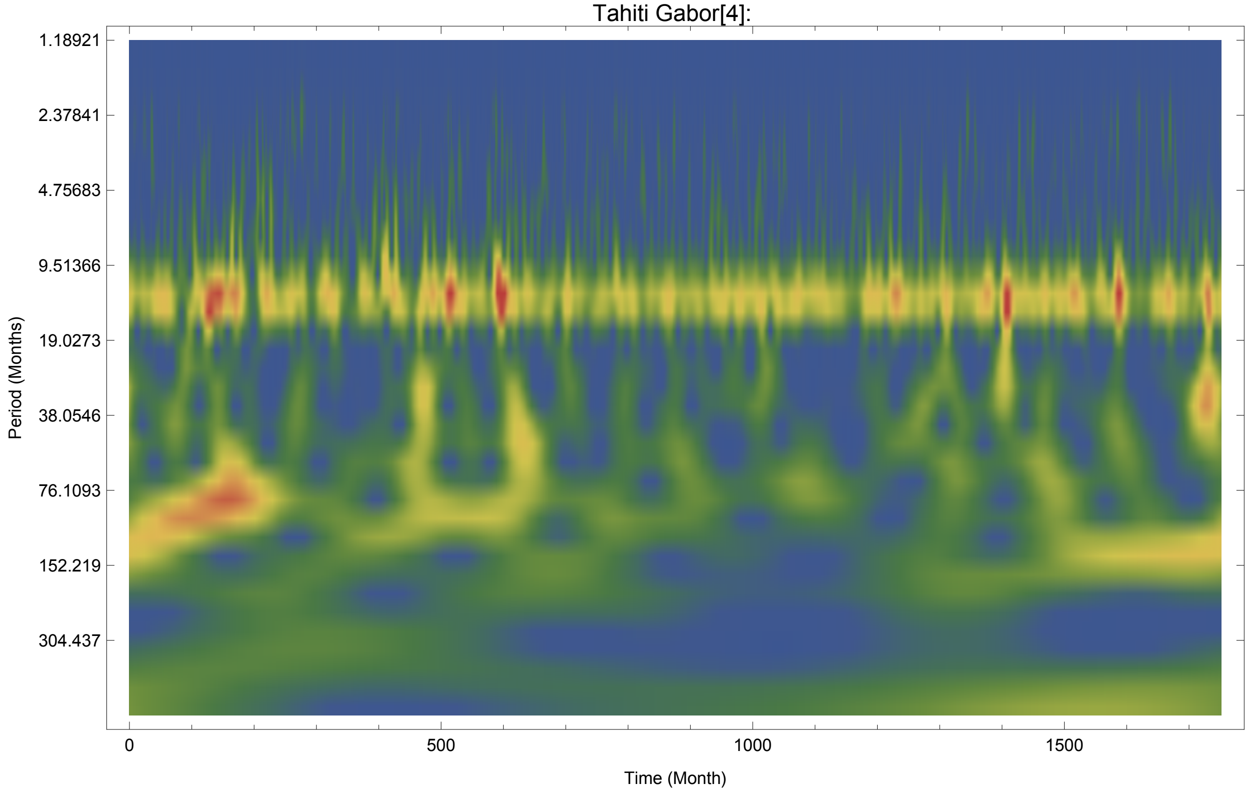


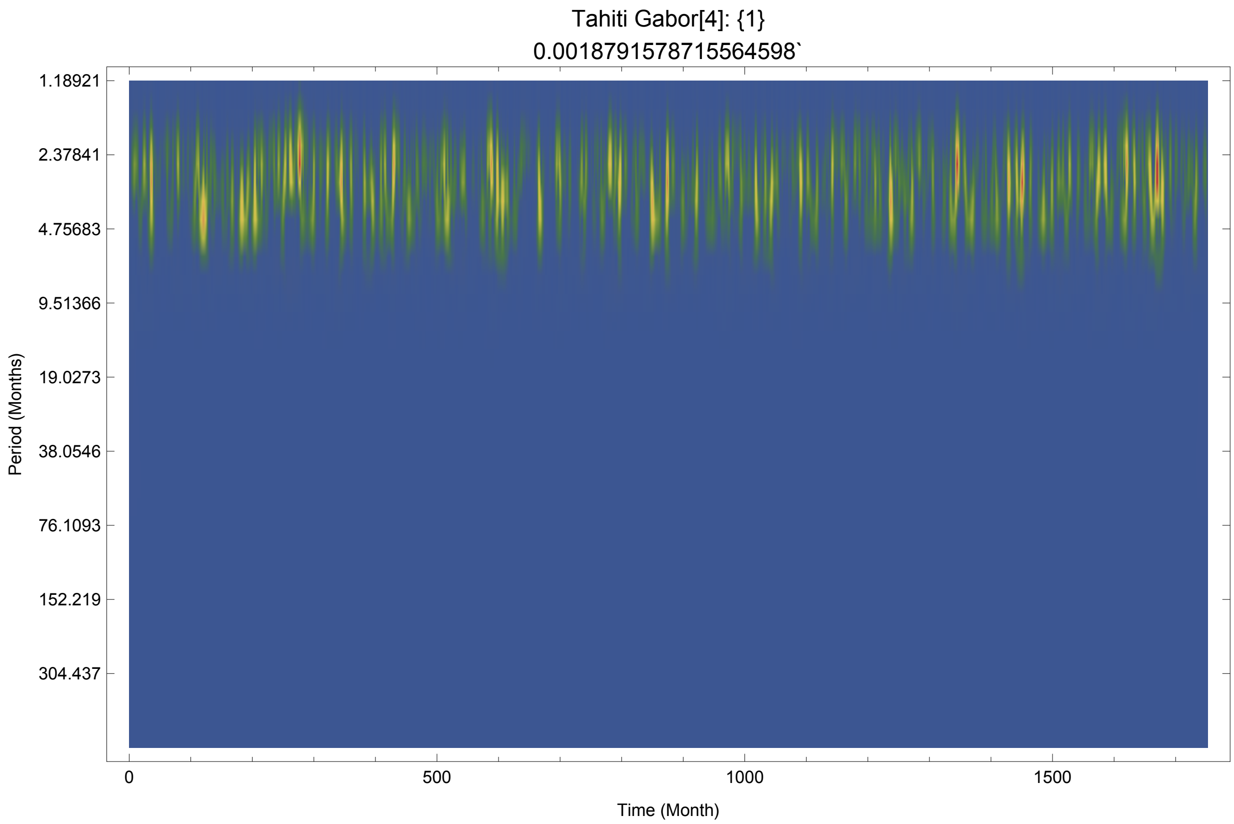
Energy Fractions show the contribution of each decomposition to the original data, like the coordinates of a decomposed vector in a normed vector space:

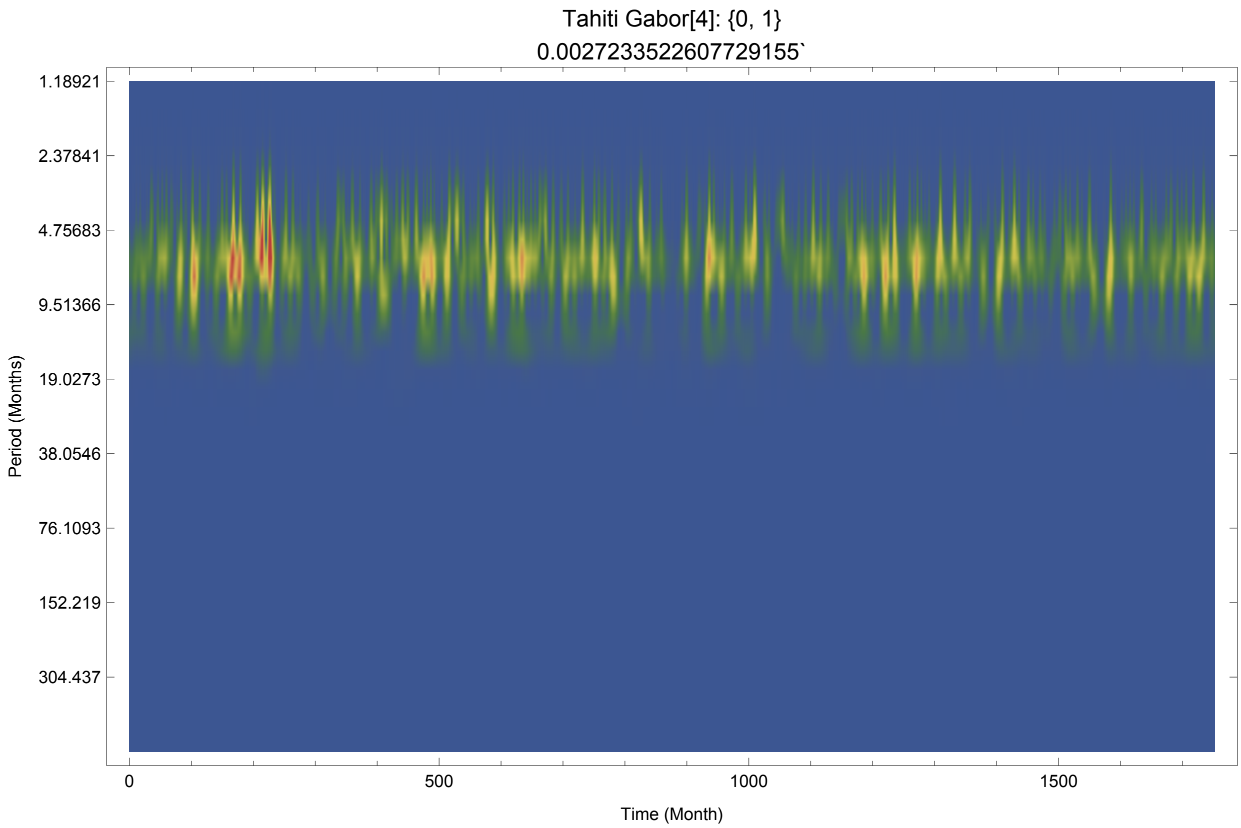
Out[10]= {{1} → 0.00187916, {0, 1} → 0.00272335,
{0, 0, 1} → 0.00861977, {0, 0, 0, 1} → 0.00159752, {0, 0, 0, 0} → 0.98518}

The Entire Data, first plot below is the entire data, then follows the Refinement indecies and their 'energy' on top of each plot, all frequencies taken into account.

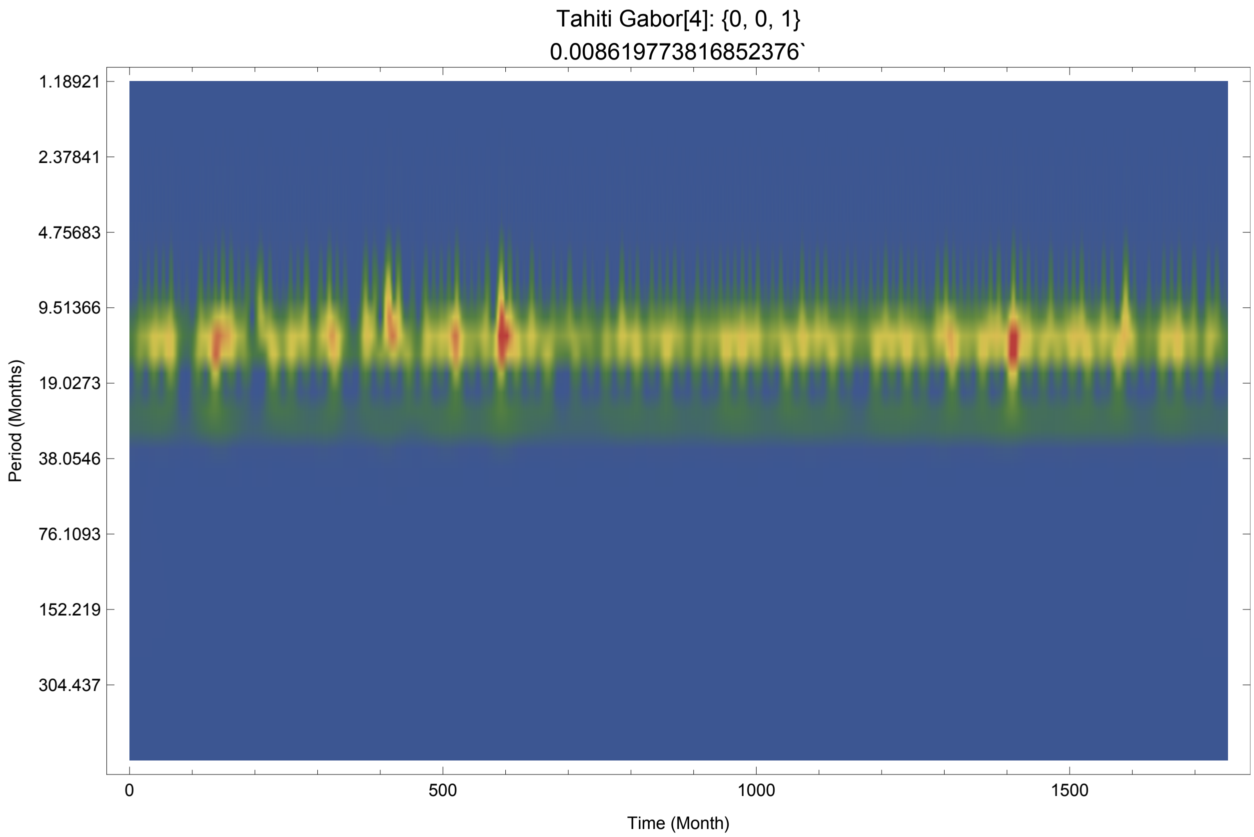
Tahiti Gabor[4]:

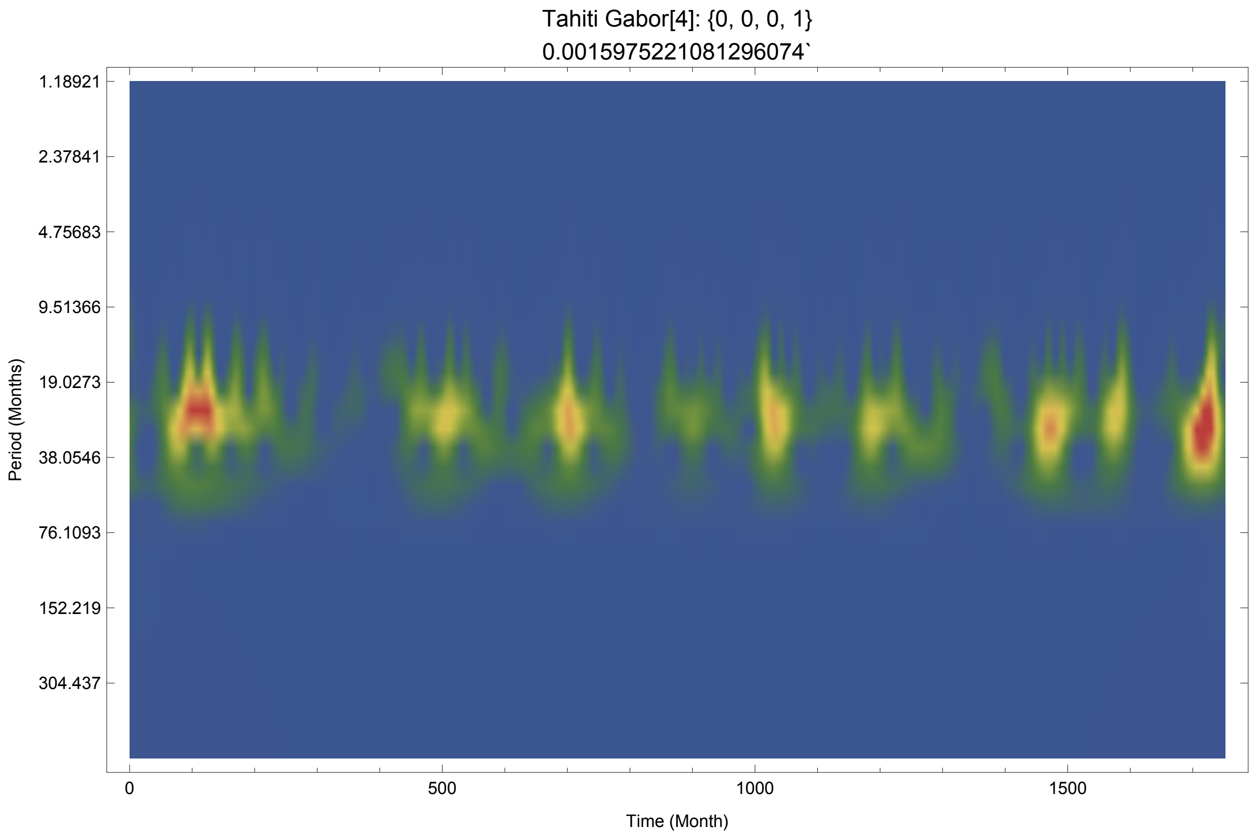



Tahiti Gabor[4]: {1}
0.0018791578715564598`
1.18921
2.37841
4.75683
9.51366
19.0273
38.0546
76.1093
152.219
304.437
Period (Months)
0
500
1000
1500
Time (Month)

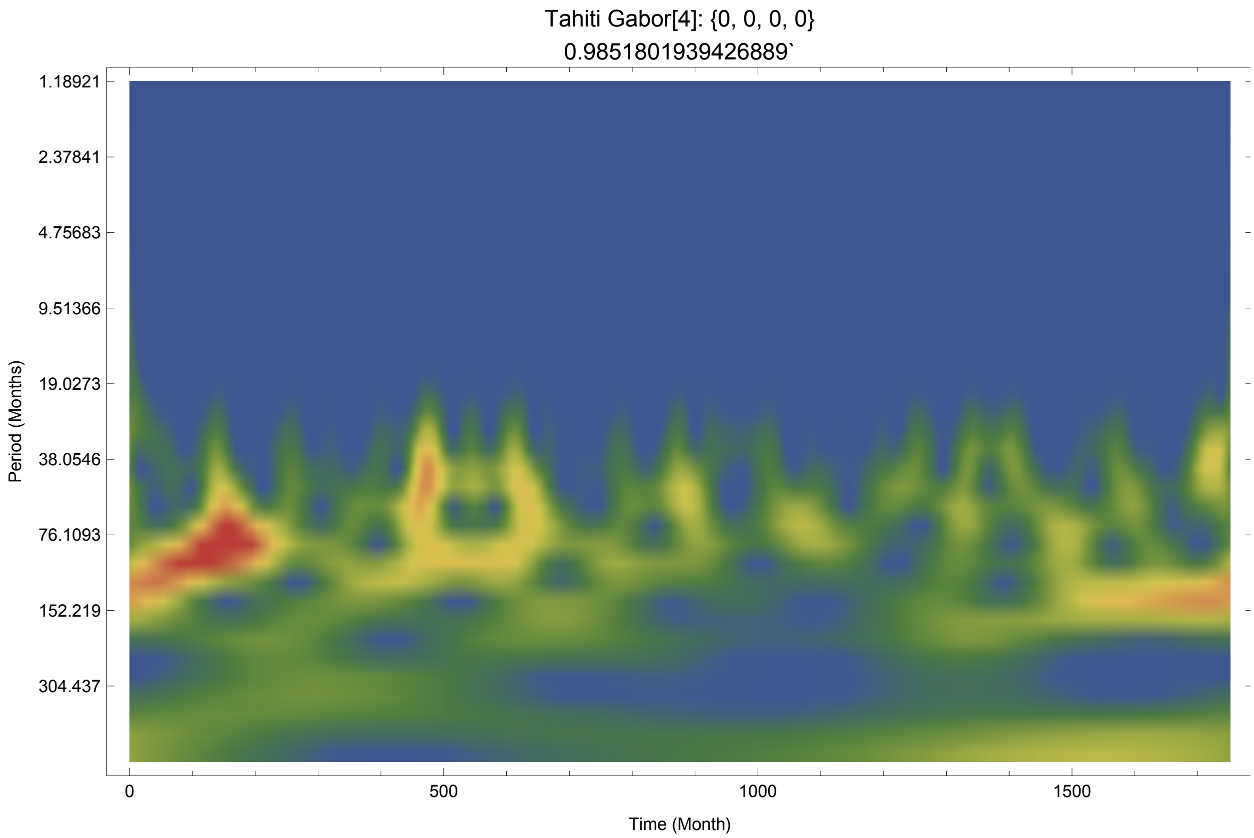

Tahiti Gabor[4]: {0, 1}
0.0027233522607729155`
1.18921
2.37841
4.75683
9.51366
19.0273
38.0546
76.1093
152.219
304.437
Period (Months)
0
500
1000
1500
Time (Month)

Out[1166]=


Tahiti Gabor[4]: {0, 0, 1}
0.008619773816852376`
1.18921
2.37841
4.75683
9.51366
19.0273
38.0546
76.1093
152.219
304.437
Period (Months)
0
500
1000
1500
Time (Month)



Tahiti Gabor[4]: {0, 0, 0, 1}
0.0015975221081296074`
1.18921
2.37841
4.75683
9.51366
19.0273
38.0546
76.1093
152.219
304.437
Period (Months)
0
500
1000
1500
Time (Month)


Tahiti Gabor[4]: {0, 0, 0, 0}
0.9851801939426889`
1.18921
2.37841
4.75683
9.51366
19.0273
38.0546
76.1093
152.219
304.437
Period (Months)
0
500
1000
1500
Time (Month)