

QBO Anomalies: Wavelet Analysis

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Source: <http://www.cru.uea.ac.uk/cru/data/temperature/HadCRUT4-gl.dat>

```
1850 -0.690 -0.279 -0.728 -0.565 -0.322 -0.215 -0.130 -0.234 -0.439 -0.455 -0.191 -0.265 -0.374
1850  22  20  18  19  18  20  22  22  23  23  24  25
1851 -0.295 -0.346 -0.468 -0.441 -0.304 -0.189 -0.216 -0.159 -0.111 -0.054 -0.021 -0.056 -0.219
1851  23  22  20  21  20  20  21  23  18  20  18  20
1852 -0.301 -0.455 -0.492 -0.561 -0.206 -0.040 -0.013 -0.205 -0.132 -0.218 -0.193 0.092 -0.223
1852  23  22  22  23  23  23  23  23  21  21  22  25
1853 -0.169 -0.324 -0.312 -0.346 -0.272 -0.176 -0.060 -0.148 -0.404 -0.363 -0.247 -0.424 -0.268
1853  25  26  26  24  25  27  26  29  28  29  26  27
1854 -0.352 -0.274 -0.271 -0.346 -0.228 -0.209 -0.230 -0.167 -0.114 -0.193 -0.366 -0.229 -0.243
```

...

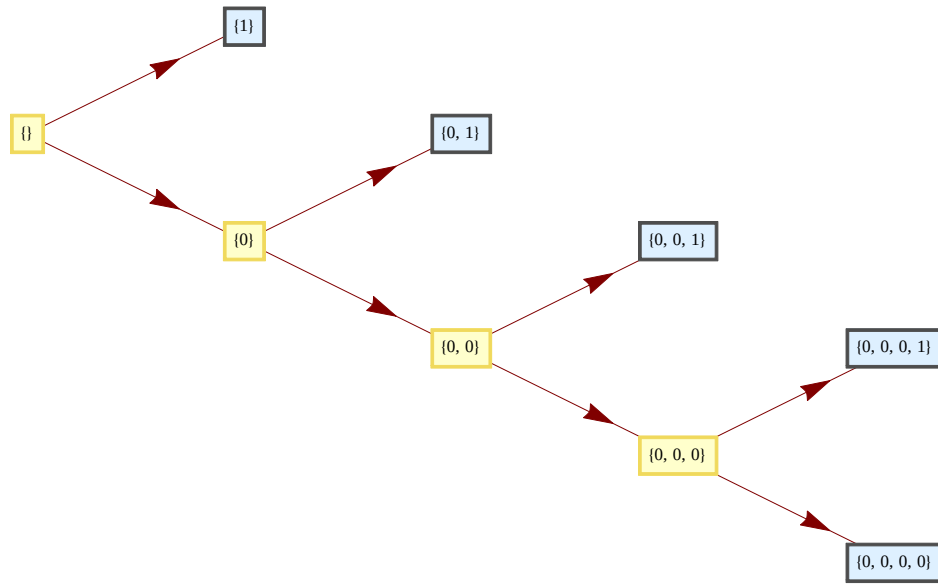
```
2010 0.545 0.558 0.668 0.676 0.582 0.582 0.607 0.531 0.423 0.484 0.576 0.328 0.547
2010  86  86  87  86  85  86  86  87  86  86  86  87
2011 0.314 0.314 0.409 0.459 0.366 0.472 0.496 0.478 0.438 0.437 0.325 0.375 0.406
2011  86  85  85  84  84  83  83  84  84  85  84  84
2012 0.300 0.276 0.353 0.535 0.543 0.518 0.487 0.537 0.534 0.526 0.518 0.268 0.448
2012  84  84  82  80  79  79  79  81  81  80  82  81
2013 0.450 0.479 0.405 0.427 0.498 0.457 0.520 0.528 0.532 0.478 0.601 0.486 0.487
2013  81  80  80  77  77  76  77  78  77  77  78  79
```

Multiple Trends

The data indicates multiple Trends, as opposed to the usual 1-Trend + Noise:

Daubechies Wavelet [4]

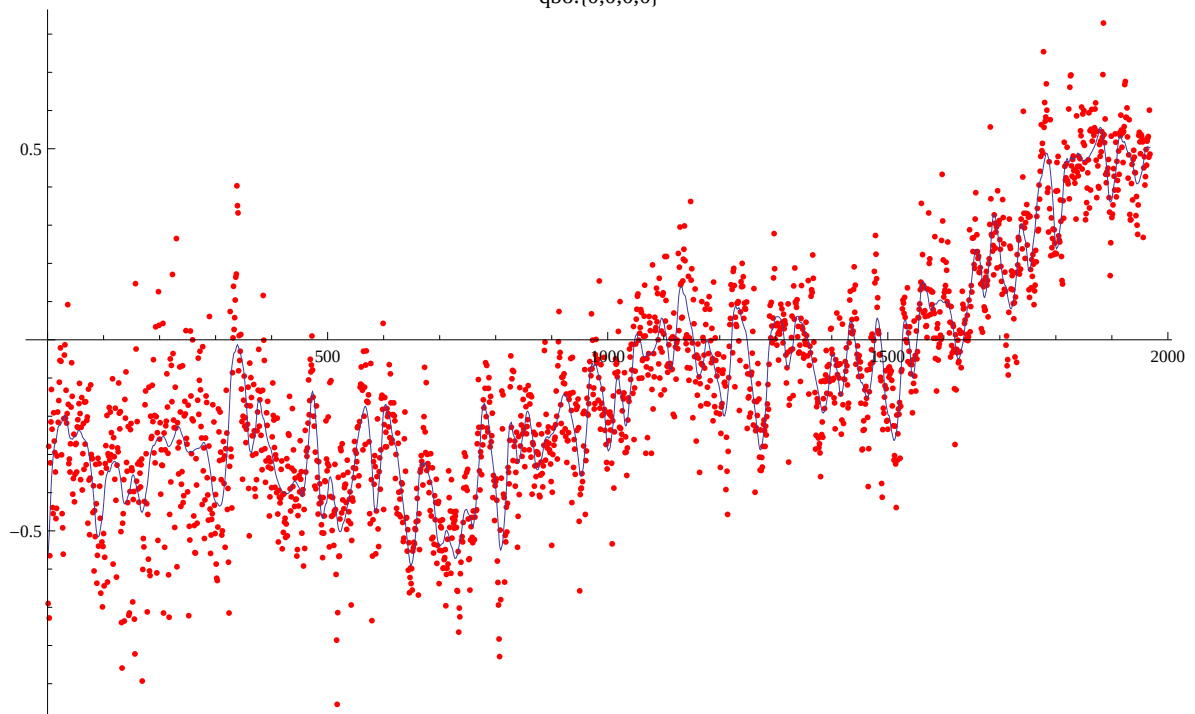
Refinement Index

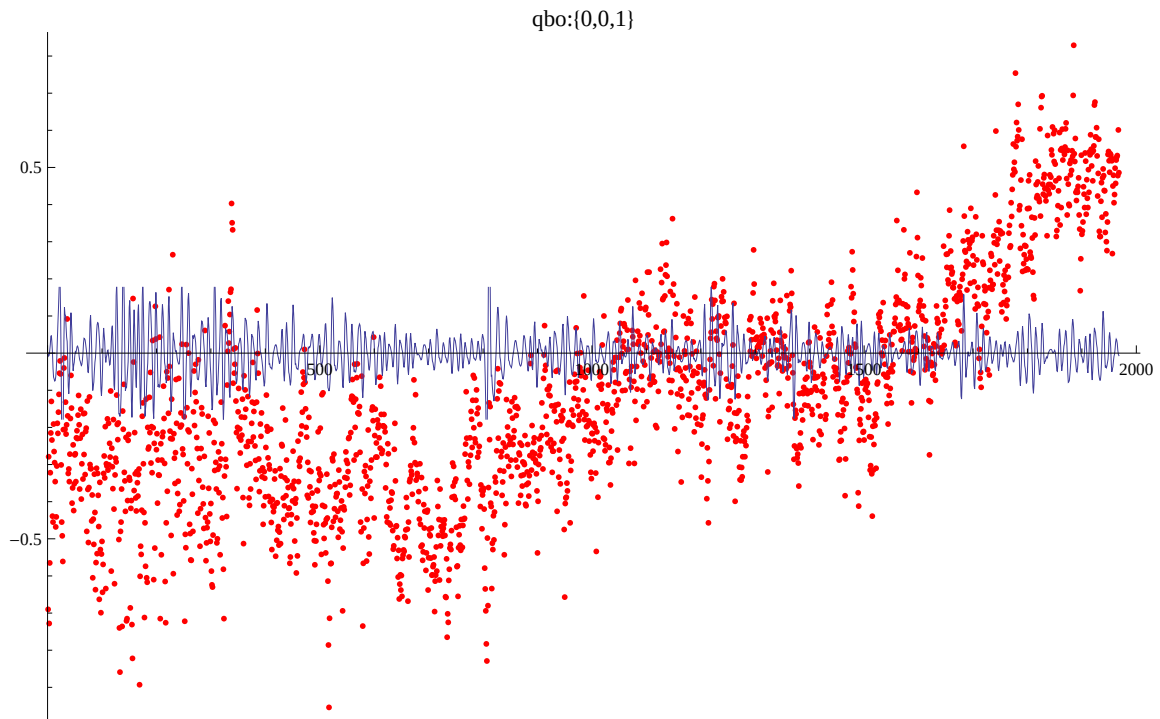
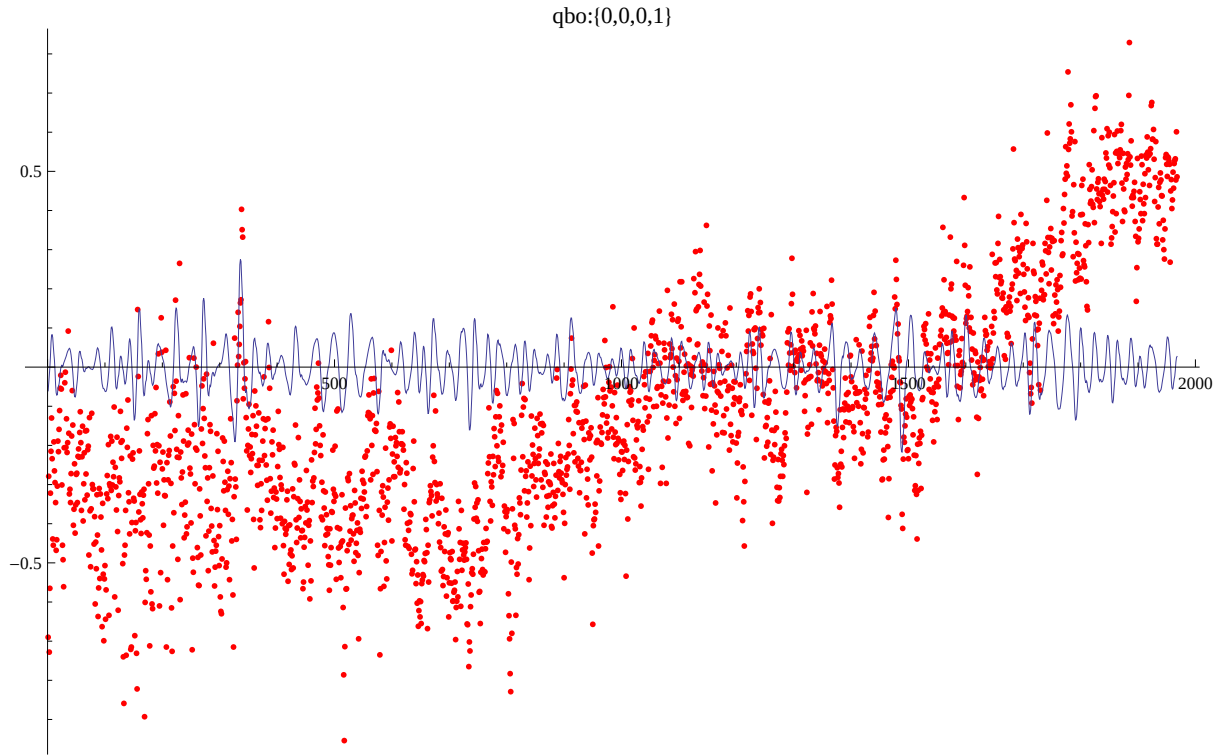


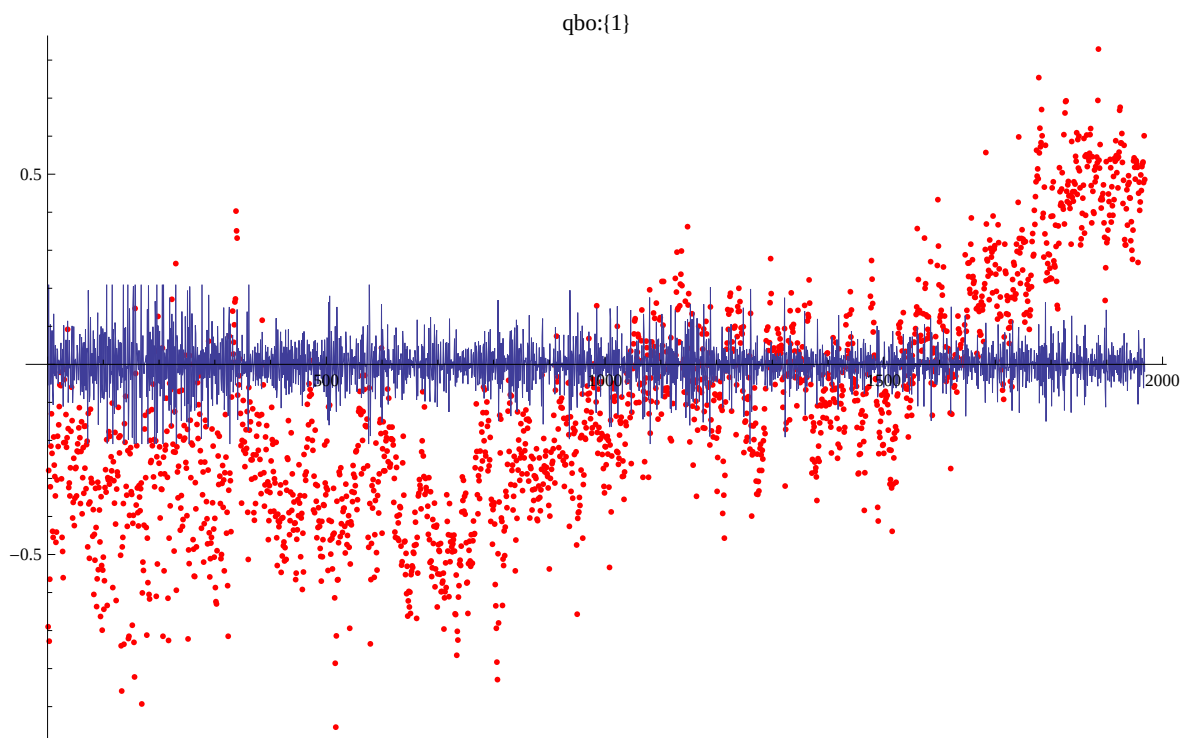
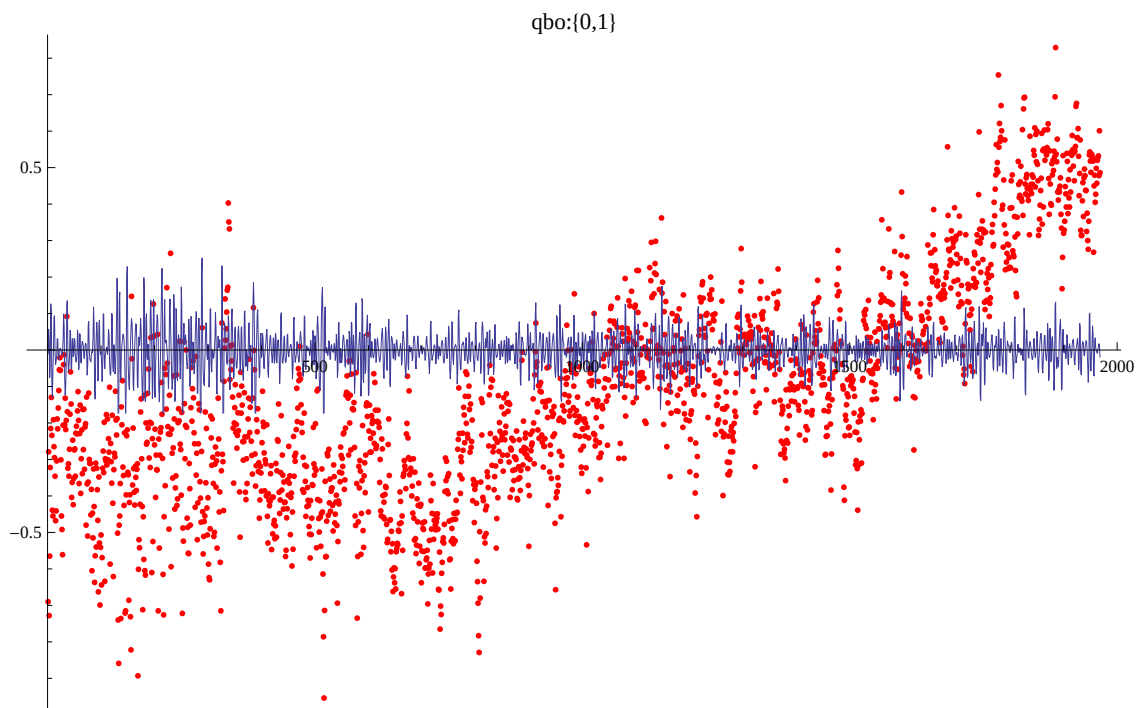
Energy Function

$\{ \{1\} \rightarrow 0.0381458, \{0, 1\} \rightarrow 0.0253344, \{0, 0, 1\} \rightarrow 0.0266079,$
 $\{0, 0, 0, 1\} \rightarrow 0.0267864, \{0, 0, 0, 0\} \rightarrow 0.883125 \}$

qbo:{0,0,0,0}







Scalograms

DGaussian & Gabor Wavelet [4] , un-padded.

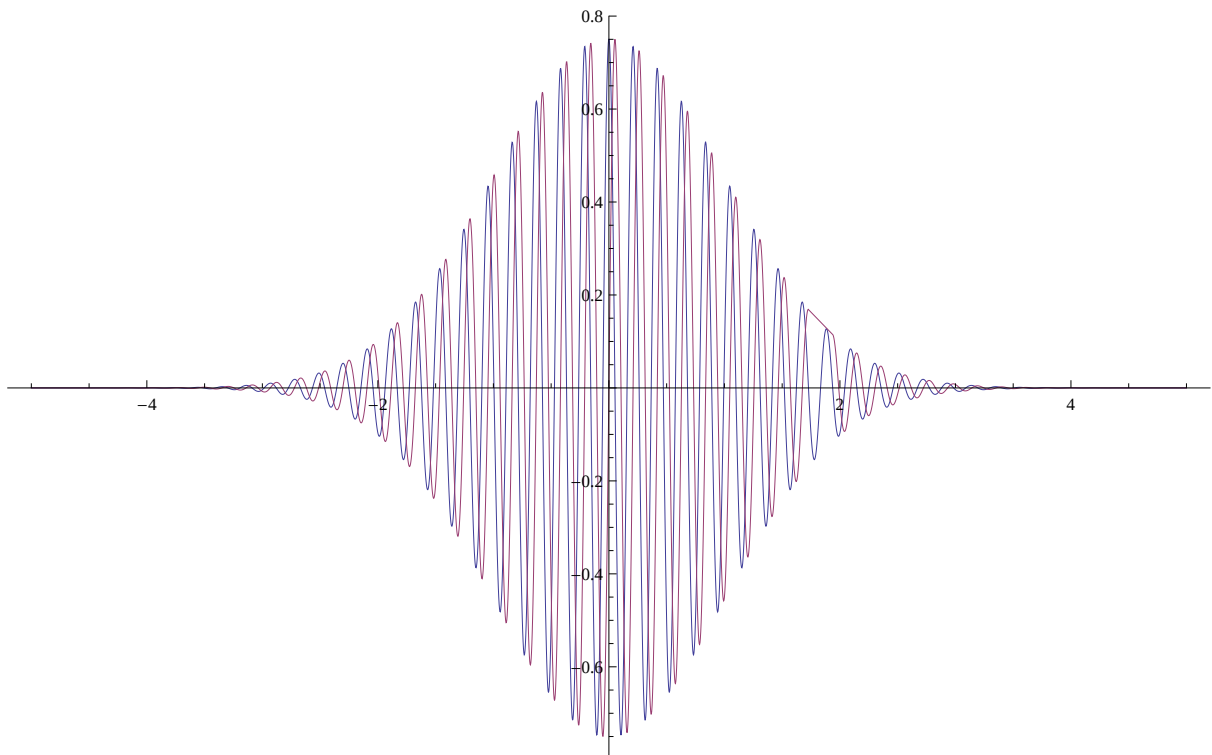
Color Scheme: Blue Min, Red Max



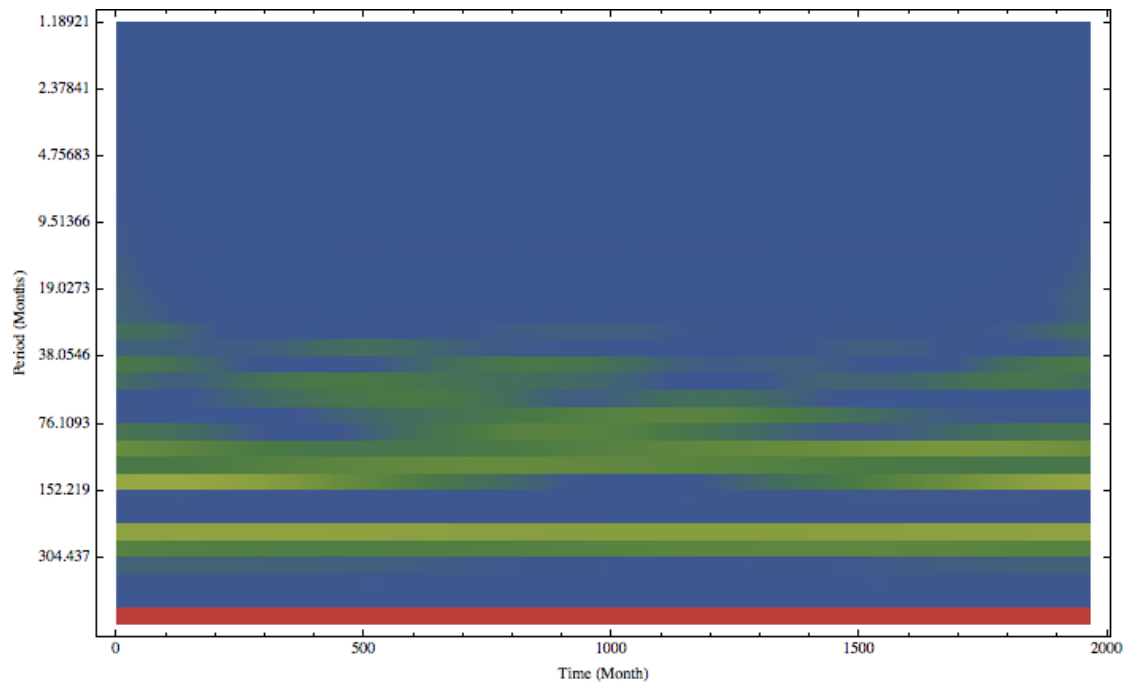
Gabor Wavelet [30]

$$\frac{1}{\sqrt[4]{\pi}} \exp(i w x) \exp\left(-\frac{x^2}{2}\right)$$

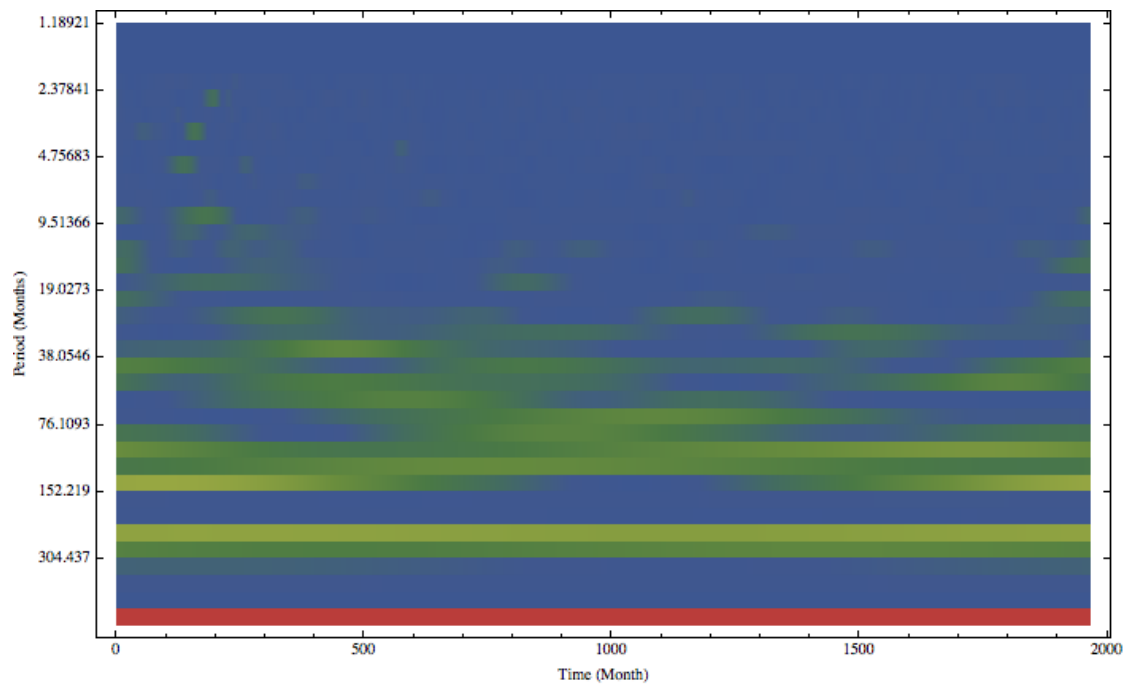
$$\frac{1}{\sqrt[4]{\pi}} \exp(i 30 x) \exp\left(-\frac{x^2}{2}\right)$$



Trend



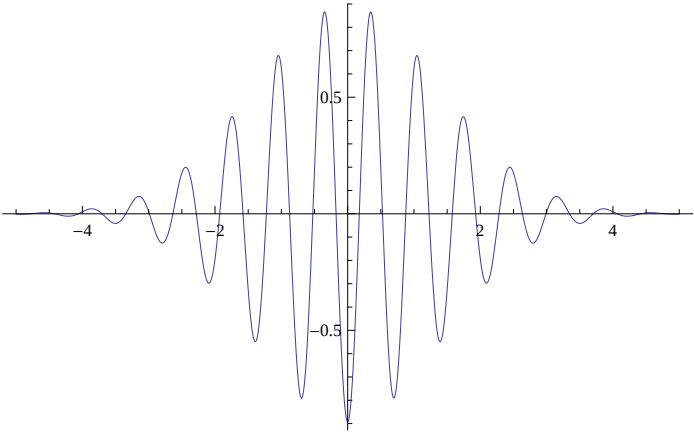
Raw Signal



DGaussian Wavelet [80]

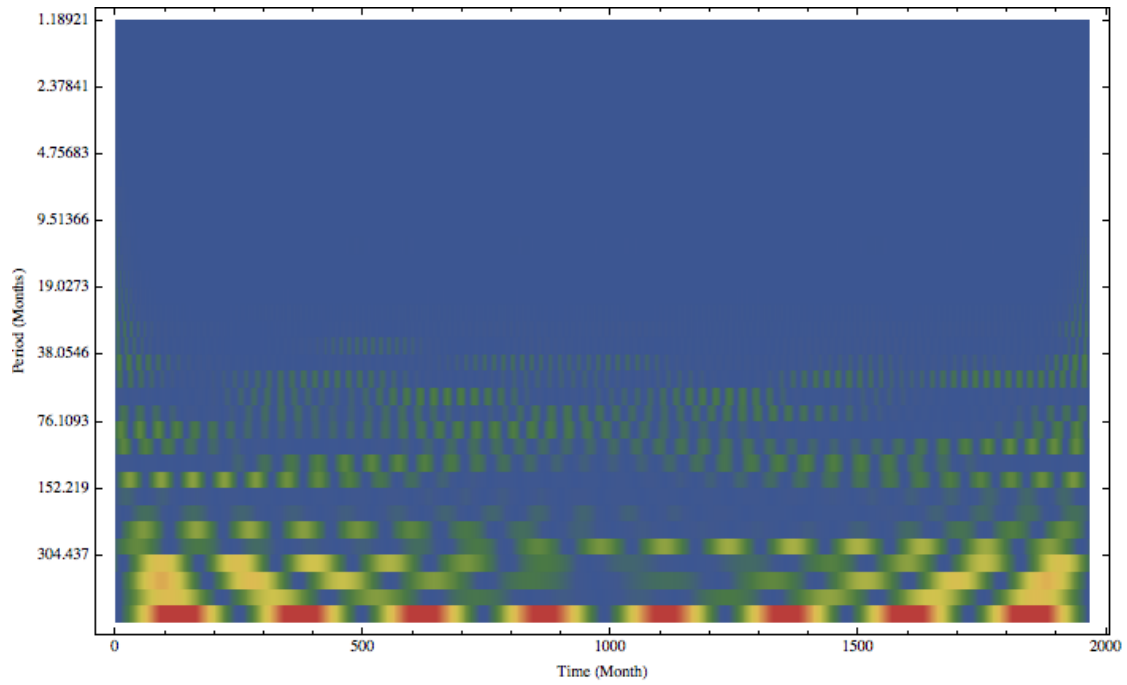
$$\frac{(-1)^{n+1}}{\sqrt{\Gamma(n+\frac{1}{2})}} \frac{\partial^n}{\partial x^n} \exp\left(-\frac{x^2}{2}\right)$$

$$\frac{(-1)^{80+1}}{\sqrt{\Gamma(80+\frac{1}{2})}} \frac{\partial^{80}}{\partial x^{80}} \exp\left(-\frac{x^2}{2}\right)$$



$$\begin{aligned}
& - \left(1\,099\,511\,627\,776 \, e^{-\frac{x^2}{2}} \right. \\
& \quad (79\,777\,941\,814\,291\,672\,401\,518\,892\,224\,505\,807\,820\,921\,910\,393\,015\,244\,140\,625 - \\
& \quad 3\,191\,117\,672\,571\,666\,896\,060\,755\,688\,980\,232\,312\,836\,876\,415\,720\,609\,765\,625\,000 \, x^2 + \\
& \quad 20\,742\,264\,871\,715\,834\,824\,394\,911\,978\,371\,510\,033\,439\,696\,702\,183\,963\,476\,562\,500 \, x^4 - \\
& \quad 52\,547\,071\,008\,346\,781\,555\,133\,777\,011\,874\,492\,084\,713\,898\,312\,199\,374\,140\,625\,000 \, x^6 + \\
& \quad 69\,437\,200\,975\,315\,389\,912\,141\,062\,479\,977\,007\,397\,657\,651\,341\,120\,601\,542\,968\,750 \, x^8 - \\
& \quad 55\,549\,760\,780\,252\,311\,929\,712\,849\,983\,981\,605\,918\,126\,121\,072\,896\,481\,234\,375\,000 \, x^{10} + \\
& \quad 29\,458\,206\,474\,376\,226\,023\,332\,571\,961\,202\,366\,774\,763\,852\,084\,111\,770\,351\,562\,500 \, x^{12} - \\
& \quad 11\,006\,362\,858\,558\,150\,382\,344\,037\,875\,614\,071\,102\,659\,021\,657\,800\,002\,109\,375\,000 \, x^{14} + \\
& \quad 3\,026\,749\,786\,103\,491\,355\,144\,610\,415\,793\,869\,553\,231\,230\,955\,895\,000\,580\,078\,125 \, x^{16} - \\
& \quad 633\,045\,706\,897\,462\,244\,213\,251\,851\,669\,306\,050\,349\,015\,624\,762\,353\,062\,500\,000 \, x^{18} + \\
& \quad 103\,286\,404\,809\,585\,945\,108\,477\,933\,693\,413\,092\,425\,365\,707\,198\,068\,131\,250\,000 \, x^{20} - \\
& \quad 13\,413\,818\,806\,439\,733\,130\,971\,160\,219\,923\,778\,237\,060\,481\,454\,294\,562\,500\,000 \, x^{22} + \\
& \quad 1\,409\,422\,990\,531\,711\,089\,848\,419\,008\,615\,179\,597\,372\,296\,964\,400\,515\,625\,000 \, x^{24} - \\
& \quad 121\,427\,211\,491\,962\,801\,586\,940\,714\,588\,384\,703\,773\,613\,276\,932\,967\,500\,000 \, x^{26} + \\
& \quad 8\,673\,372\,249\,425\,914\,399\,067\,193\,899\,170\,335\,983\,829\,519\,780\,926\,250\,000 \, x^{28} - \\
& \quad 518\,408\,456\,287\,525\,918\,105\,165\,612\,364\,203\,989\,838\,086\,239\,779\,500\,000 \, x^{30} + \\
& \quad 26\,129\,458\,482\,234\,169\,259\,332\,944\,171\,582\,862\,391\,032\,572\,569\,531\,250 \, x^{32} - \\
& \quad 1\,117\,837\,796\,031\,408\,310\,559\,698\,146\,377\,876\,465\,926\,527\,168\,750\,000 \, x^{34} + \\
& \quad 40\,809\,951\,283\,686\,335\,147\,417\,551\,375\,700\,251\,930\,650\,991\,875\,000 \, x^{36} - \\
& \quad 1\,277\,125\,075\,734\,138\,511\,014\,489\,516\,735\,996\,504\,230\,898\,750\,000 \, x^{38} + \\
& \quad 34\,384\,136\,654\,380\,652\,219\,620\,871\,604\,430\,675\,113\,908\,812\,500 \, x^{40} - \\
& \quad 798\,702\,361\,309\,655\,103\,823\,945\,914\,156\,345\,531\,101\,250\,000 \, x^{42} + \\
& \quad 16\,041\,590\,766\,261\,571\,852\,700\,816\,457\,685\,586\,776\,875\,000 \, x^{44} - \\
& \quad 278\,984\,187\,239\,331\,684\,394\,796\,807\,959\,749\,335\,250\,000 \, x^{46} + \\
& \quad 4\,204\,548\,921\,160\,140\,633\,609\,526\,361\,095\,513\,031\,250 \, x^{48} - \\
& \quad 54\,916\,557\,337\,601\,836\,847\,144\,834\,104\,104\,660\,000 \, x^{50} + \\
& \quad 621\,228\,024\,181\,016\,253\,926\,977\,761\,358\,650\,000 \, x^{52} - \\
& \quad 6\,077\,702\,542\,651\,451\,820\,389\,719\,538\,100\,000 \, x^{54} + \\
& \quad 51\,305\,281\,204\,200\,567\,314\,978\,151\,945\,000 \, x^{56} - \\
& \quad 372\,452\,132\,153\,906\,114\,809\,278\,780\,000 \, x^{58} + 2\,314\,674\,267\,623\,145\,346\,272\,354\,000 \, x^{60} - \\
& \quad 12\,240\,477\,353\,903\,465\,606\,940\,000 \, x^{62} + 54\,644\,988\,187\,069\,042\,888\,125 \, x^{64} - \\
& \quad 203\,804\,151\,746\,644\,449\,000 \, x^{66} + 626\,263\,855\,235\,518\,500 \, x^{68} - 1\,555\,935\,044\,063\,400 \, x^{70} + \\
& \quad 3\,043\,691\,400\,750 \, x^{72} - 4\,507\,503\,000 \, x^{74} + 4\,744\,740 \, x^{76} - 3160 \, x^{78} + x^{80}) \Big) / \\
& \quad (45\,168\,379\,707\,306\,661\,756\,751\,999\,941\,371\,251\,562\,392\,578\,125 \\
& \quad \sqrt{2\,670\,913\,613\,325\,913\,901\,936\,358\,283\,331\,289\,650\,302\,990\,048\,697} \, \pi^{1/4})
\end{aligned}$$

Trend



Raw Signal

