Let's define the Poisson Bracket as in the math books and modify the symbols { } so they become some sort of operator:

In[1]:=

```
<< Notation`
```

In[69]:=

```
(* Poisson Bracket defined as function and as symbolic operator *)
Notation[ {x_, y_}Full ⟹ poissonBra[x_, y_] ]

makeMatrix[dim_, a_] := Array[Subscript[a, #1, #2] &, dim]
makeVar[dim_, a_] := Array[Subscript[a, #1] &, dim]
poissonBra[xx_, yy_] := Module[{},

  Sum[((∂q_i xx) * (∂p_i yy) - (∂p_i xx) * (∂q_i yy)), {i, 1, n}]
 ]

Notation[ {x_, y_}New ⟹ makeNewBra[x_, y_] ]
Notation[ H_i_ ⟹ H[[i_]] ]

makeF[] := Module[{},

  Table[Table[(∂u_i H_j) - (∂u_j H_i) + {H_j, H_i}Full, {i, 1, M}],
   {j, 1, M}]
 ]

makeNewBra[x_, y_] := Module[{v1, v2},

  v1 = Table[(∂u_i x) + {x, H_i}Full, {i, 1, M}];
  v2 = Table[(∂u_i y) + {y, H_i}Full, {i, 1, M}];


  {x, y}Full + v1.G.v2


 ]
```

For this draft of the code I assume n = 4, M = 2. <u>If you need the general such formal system with n and m arbitrary, I have to consult some experts in Boston to get back to you</u>.

At lease we get an idea how the symbolic mechanisms work.

We can simply change n and m below to any numbers and all the code works without modification!

In[86]:=
```
n = 3;
M = 2;
f =.;
g =.;
h =.;
makeVar[n, p]
makeVar[n, q]
makeVar[M, u]
```

Out[91]= $\{p_1, p_2, p_3\}$

Out[92]= $\{q_1, q_2, q_3\}$

Out[93]= $\{u_1, u_2\}$

The concrete functional dependence is not important. Only for any case - all functions depend on mutually independent arguments (q_i, p_i, q_α). The definition of Poisson bracket is

In[102]:=
```
X := f[p1, p2, p3 , q1, q2, q3, u1, u2] ;
Y := g[p1, p2, p3 , q1, q2, q3, u1, u2] ;
Z := z[p1, p2, p3 , q1, q2, q3, u1, u2] ;

(* vector of functions *)
H = Table[hi[p1, p2, p3 , q1, q2, q3, u1, u2], {i, 1, M}]
```

Out[105]= $\{h_1[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2], h_2[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\}$

H indexing to match the paper:

In[106]:= H_1

Out[106]= $h_1[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$

In[107]:=
H_2

Out[107]= $h_2[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$

Let’s mak F:

In[108]:=
```
F = makeF[];

(*Let's check for the enteries, too long to output properly *)
F[[1]][[1]]
```

Out[109]= 0

In[110]:=
```
F[[1]][[2]]
```

Out[110]=

$$h_1{}^{(0,0,0,0,0,0,0,1)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$$
$$h_2{}^{(0,0,0,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$$
$$h_2{}^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_1{}^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$$
$$h_1{}^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_2{}^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$$
$$h_2{}^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_1{}^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$$
$$h_1{}^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_2{}^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$$
$$h_2{}^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_1{}^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$$
$$h_1{}^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_2{}^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$

In[111]:=
```
F[[2]][[1]]
```

Out[111]=

$$-h_1{}^{(0,0,0,0,0,0,0,1)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$$
$$h_2{}^{(0,0,0,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$$
$$h_2{}^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_1{}^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$$
$$h_1{}^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_2{}^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$$
$$h_2{}^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_1{}^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$$
$$h_1{}^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_2{}^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$$
$$h_2{}^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_1{}^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$$
$$h_1{}^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_2{}^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$

In[113]:=
```
F[[2]][[2]]
```

Out[113]= 0

For experimentation make G inverse of F:

In[114]:=

```
G = Inverse[F];
Dimensions[G]
```

Out[115]= {2, 2}

In[116]:=

```
G[[1]][[1]]
```

Out[116]= 0

In[117]:= `G[[1]][[2]]`

Out[117]=
$$-1\Big/\Big(h_1^{(0,0,0,0,0,0,0,1)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]-h_2^{(0,0,0,0,0,0,1,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]-h_2^{(0,0,0,0,0,1,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\,h_1^{(0,0,1,0,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]+h_1^{(0,0,0,0,0,1,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\,h_2^{(0,0,1,0,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]-h_2^{(0,0,0,0,1,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\,h_1^{(0,1,0,0,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]+h_1^{(0,0,0,0,1,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\,h_2^{(0,1,0,0,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]-h_2^{(0,0,0,1,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\,h_1^{(1,0,0,0,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]+h_1^{(0,0,0,1,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\,h_2^{(1,0,0,0,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\Big)$$

In[118]:=

```
G[[2]][[1]]
```

Out[118]=
$$-1\Big/\Big(-h_1^{(0,0,0,0,0,0,0,1)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]+h_2^{(0,0,0,0,0,0,1,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]+h_2^{(0,0,0,0,0,1,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\,h_1^{(0,0,1,0,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]-h_1^{(0,0,0,0,0,1,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\,h_2^{(0,0,1,0,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]+h_2^{(0,0,0,0,1,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\,h_1^{(0,1,0,0,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]-h_1^{(0,0,0,0,1,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\,h_2^{(0,1,0,0,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]+h_2^{(0,0,0,1,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\,h_1^{(1,0,0,0,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]-h_1^{(0,0,0,1,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\,h_2^{(1,0,0,0,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\Big)$$

In[119]:=

```
G[[2]][[2]]
```

Out[119]= 0

In[120]:=

```
new = {X, Y}_New ;
Dimensions[new]
```

Out[121]= {8}

In[122]:= `new`

Out[122]=
$$-g^{(0,0,0,0,0,1,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]\,f^{(0,0,1,0,0,0,0,0)}[p_1,p_2,p_3,q_1,q_2,q_3,u_1,u_2]+$$

$f^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$g^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$
$g^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$f^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$
$f^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$g^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$
$g^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$f^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$
$f^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$g^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$
$\Big(\Big(g^{(0,0,0,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - h_1^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3,$
$u_1, u_2]\, g^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + g^{(0,0,0,0,0,1,0,0)}[p_1,$
$p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, h_1^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$
$h_1^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, g^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1,$
$q_2, q_3, u_1, u_2] + g^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, h_1^{(0,1,0,0,0,0,0,0)}[$
$p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - h_1^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$g^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + g^{(0,0,0,1,0,0,0,0)}[p_1, p_2,$
$p_3, q_1, q_2, q_3, u_1, u_2]\, h_1^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\Big)$
$\Big(f^{(0,0,0,0,0,0,0,1)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - h_2^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3,$
$q_1, q_2, q_3, u_1, u_2]\, f^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + f^{(0,0,0,0,0,1,0,0)}[$
$p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, h_2^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$
$h_2^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, f^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1,$
$q_2, q_3, u_1, u_2] + f^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, h_2^{(0,1,0,0,0,0,0,0)}[$
$p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - h_2^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$f^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + f^{(0,0,0,1,0,0,0,0)}[p_1, p_2,$
$p_3, q_1, q_2, q_3, u_1, u_2]\, h_2^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\Big)\Big) \Big/$
$\Big(-h_1^{(0,0,0,0,0,0,0,1)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + h_2^{(0,0,0,0,0,0,1,0)}[p_1, p_2,$
$p_3, q_1, q_2, q_3, u_1, u_2] + h_2^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$h_1^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$
$h_1^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$h_2^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$
$h_2^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$h_1^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$
$h_1^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$h_2^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$
$h_2^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$h_1^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$
$h_1^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$h_2^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\Big) -$
$\Big(\Big(f^{(0,0,0,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - h_1^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1,$
$q_2, q_3, u_1, u_2]\, f^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + f^{(0,0,0,0,0,1,0,0)}[$
$p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, h_1^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$
$h_1^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, f^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1,$
$q_2, q_3, u_1, u_2] + f^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, h_1^{(0,1,0,0,0,0,0,0)}[$
$p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - h_1^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$

$$f^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + f^{(0,0,0,1,0,0,0,0)}[p_1, p_2,$$
$$p_3, q_1, q_2, q_3, u_1, u_2]\, h_1^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\Big)$$
$$\Big(g^{(0,0,0,0,0,0,0,1)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - h_2^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3,$$
$$q_1, q_2, q_3, u_1, u_2]\, g^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + g^{(0,0,0,0,0,1,0,0)}[$$
$$p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, h_2^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$$
$$h_2^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, g^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1,$$
$$q_2, q_3, u_1, u_2] + g^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, h_2^{(0,1,0,0,0,0,0,0)}[$$
$$p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - h_2^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$g^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + g^{(0,0,0,1,0,0,0,0)}[p_1, p_2,$$
$$p_3, q_1, q_2, q_3, u_1, u_2]\, h_2^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\Big)\Big) \Big/$$
$$\Big(h_1^{(0,0,0,0,0,0,0,1)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - h_2^{(0,0,0,0,0,0,1,0)}[p_1, p_2, p_3,$$
$$q_1, q_2, q_3, u_1, u_2] - h_2^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_1^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$$
$$h_1^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_2^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$$
$$h_2^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_1^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$$
$$h_1^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_2^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$$
$$h_2^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_1^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$$
$$h_1^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$
$$h_2^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\Big)$$

Very large, so was placed in a separate file:

$$\{\{\mathbf{X}, \mathbf{Y}\}_{\text{New}}, \mathbf{Z}\}_{\text{New}} + \{\mathbf{Y}, \{\mathbf{X}, \mathbf{Z}\}_{\text{New}}\}_{\text{New}} - \{\mathbf{X}, \{\mathbf{Y}, \mathbf{Z}\}_{\text{New}}\}_{\text{New}}$$

$Aborted[]

Let’s see how the Poisson Bracket looks like:

$$\{X, Y\} = \frac{\partial X}{\partial q_i}\frac{\partial Y}{\partial p_i} - \frac{\partial Y}{\partial q_i}\frac{\partial X}{\partial p_i}.$$

In[124]:=

$\{X, Y\}_{Full}$

Out[124]= $-g^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$f^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$
$f^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$g^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$
$g^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$f^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$
$f^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$g^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] -$
$g^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$
$f^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] +$
$f^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, g^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$

Check the anti-symmetic property:

$$\{X, Y\} = -\{Y, X\} \quad \text{antisymmentric Poisson bracket}$$

In[125]:=

$\{X, Y\}_{Full} + \{Y, X\}_{Full}$

Out[125]= 0

$$\frac{\partial}{\partial q_\alpha} \Longrightarrow \partial_\alpha,$$

In[126]:= $\partial_{u_1} X$

Out[126]= $f^{(0,0,0,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$

In[127]:=

$\partial_{u_1} Y$

Out[127]= $g^{(0,0,0,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$

In[128]:=

$\partial_{u_1}\{X, Y\}_{Full}$

Out[128]=

$$-g^{(0,0,0,0,0,1,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, f^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + f^{(0,0,0,0,0,1,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, g^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - g^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, f^{(0,0,1,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + f^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, g^{(0,0,1,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - g^{(0,0,0,0,1,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, f^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + f^{(0,0,0,0,1,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, g^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - g^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, f^{(0,1,0,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + f^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, g^{(0,1,0,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - g^{(0,0,0,1,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, f^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + f^{(0,0,0,1,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, g^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - g^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, f^{(1,0,0,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + f^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, g^{(1,0,0,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]$$

In[129]:=

```
{∂u1 X, Y}Full + {X, ∂u1 Y}Full
```

Out[129]=

$$
\begin{aligned}
&-g^{(0,0,0,0,0,1,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] \\
&\quad f^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + \\
&f^{(0,0,0,0,0,1,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] \\
&\quad g^{(0,0,1,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - \\
&g^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] \\
&\quad f^{(0,0,1,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + \\
&f^{(0,0,0,0,0,1,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] \\
&\quad g^{(0,0,1,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - \\
&g^{(0,0,0,0,1,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] \\
&\quad f^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + \\
&f^{(0,0,0,0,1,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] \\
&\quad g^{(0,1,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - \\
&g^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] \\
&\quad f^{(0,1,0,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + \\
&f^{(0,0,0,0,1,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] \\
&\quad g^{(0,1,0,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - \\
&g^{(0,0,0,1,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] \\
&\quad f^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + \\
&f^{(0,0,0,1,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] \\
&\quad g^{(1,0,0,0,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] - \\
&g^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] \\
&\quad f^{(1,0,0,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2] + \\
&f^{(0,0,0,1,0,0,0,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]\, g^{(1,0,0,0,0,0,1,0)}[p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2]
\end{aligned}
$$

$$\partial_\alpha \{X, Y\} = \{\partial_\alpha X, Y\} + \{X, \partial_\alpha Y\}$$

In[130]:=

```
∂u1 {X, Y}Full === {∂u1 X, Y}Full + {X, ∂u1 Y}Full
```

Out[130]= True

Let's go thru all parameters and see if the latter is correct, and print True each time:

In[131]:=

```
Table[∂ui {X, Y}Full === {∂ui X, Y}Full + {X, ∂ui Y}Full, {i, 1, M}]
```

Out[131]= {True, True}

In[132]:=

```
Table[∂vi {X, Y}Full === {∂vi X, Y}Full + {X, ∂vi Y}Full, {i, 1, M}]
```

Out[132]= {True, True}

The standard jacobi identity for the Poisson bracket $\{\_, \_\}$ is

$$\{\{X, Y\}, Z\} = \{X, \{Y, Z\}\} - \{Y, \{X, Z\}\}.$$

In[133]:=

```
(* Expand is a function that distributes all terms and undoes the factors *)
Expand[{{X, Y}_Full, Z}_Full + {Y, {X, Z}_Full}_Full - {X,  {Y, Z}_Full}_Full]
```

Out[133]= 0