

SVR Δ -Forecast for El Nino 3.4 Anomalies

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Data

1950	1	24.84	26.26	-1.42
1950	2	25.22	26.53	-1.31
1950	3	26.04	27.09	-1.04
...				
2013	10	26.65	26.79	-0.14
2013	11	26.54	26.74	-0.20
2013	12	26.20	26.69	-0.49



$$\Delta_t = X_t - X_{t-1}$$

Example: let t be 1950,2 then

$$\Delta_{t=1950,2} = -1.31 - (-1.42) = 0.11$$

2013, 12:

$$\Delta_{t=2013,12} = -0.49 - (-0.20) = -0.29$$

0.11
0.27
-0.08

...
0.14
-0.06
-0.29

Forecast Model

$$X_{t+1} = X_t + \text{Forecast}(\Delta_t)$$

$\Delta_{t+1} \approx \text{Forecast}(\Delta_t)$ since:

$$\Delta_{t+1} = X_{t+1} - X_t \implies X_{t+1} = X_t + \Delta_{t+1}$$

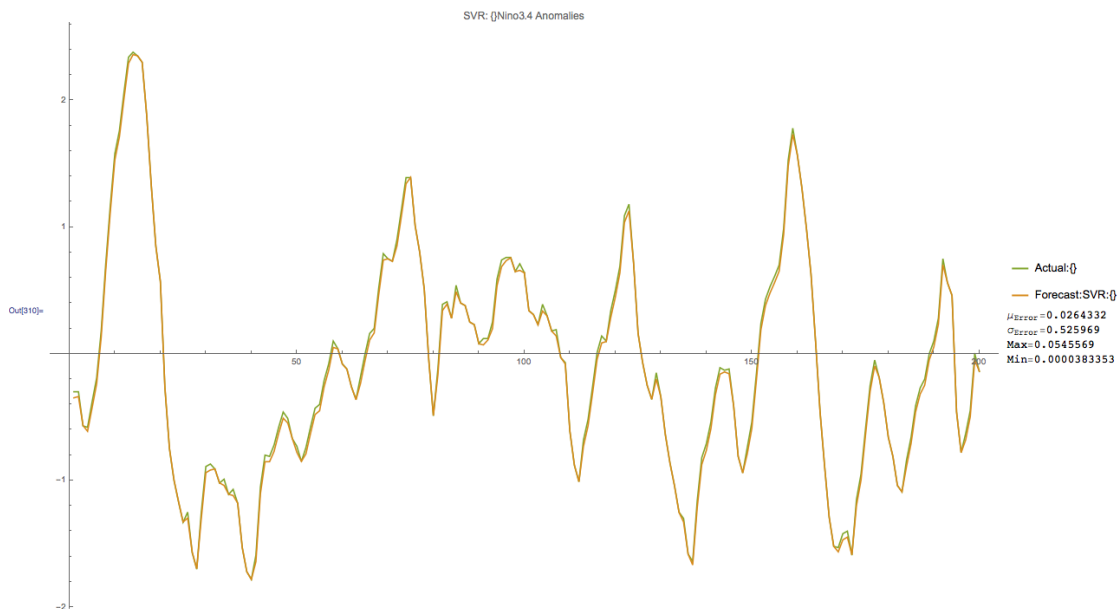
This model avoids wobbling and oscillation and reduces the over-fitting but prone to noise. For noisy data Wavelet Transforms which remove the noise create near ideal forecast for Trend.

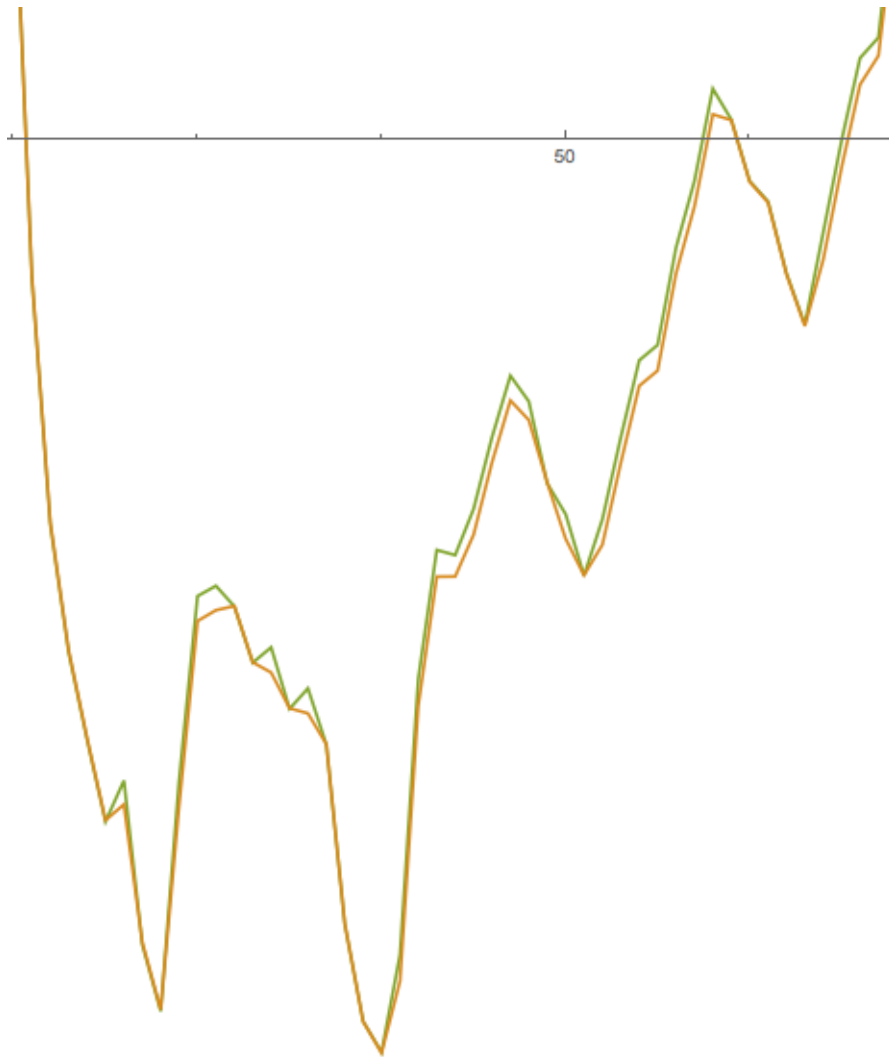
SVR Fit

Remark 1. $\{\}$ stands for raw data untransformed with no filters

Normalized error average of 1%.

Note that this is the Fit accuracy, the forecast accuracy is far less accurate.

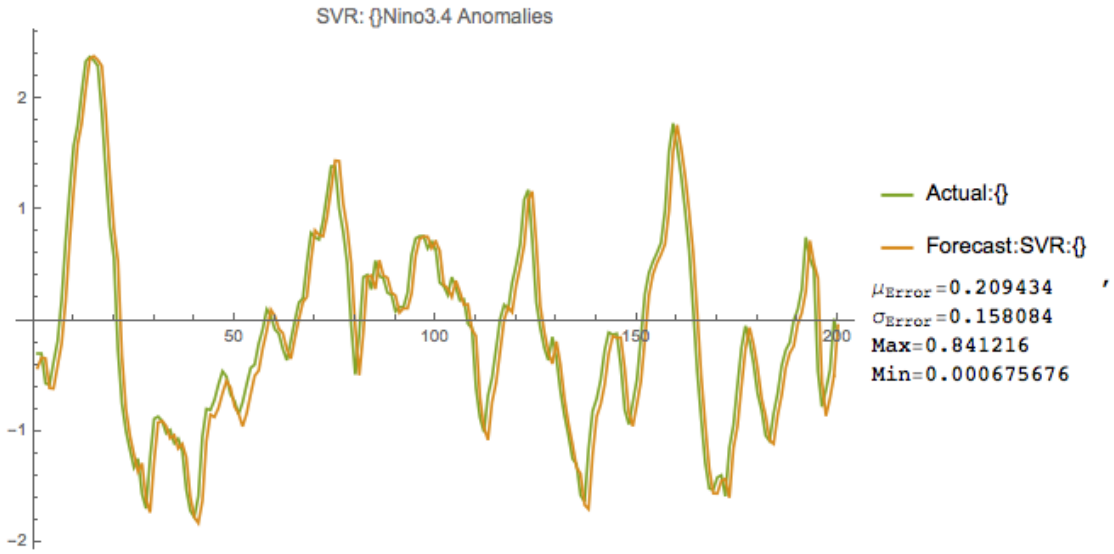




SVR Forecast

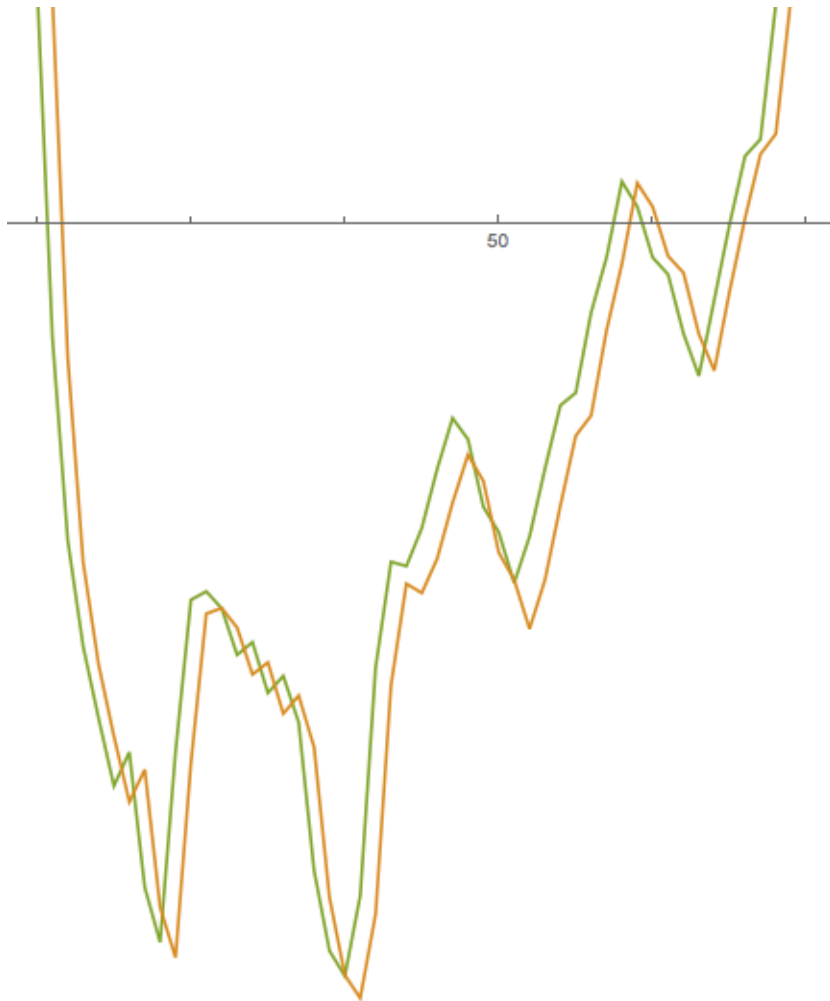
Average error of about 9% with maximum deviation of 35%.

Remark 2. For normalization divide μ_{Error} by 2.38 signal maximum.



Zoom

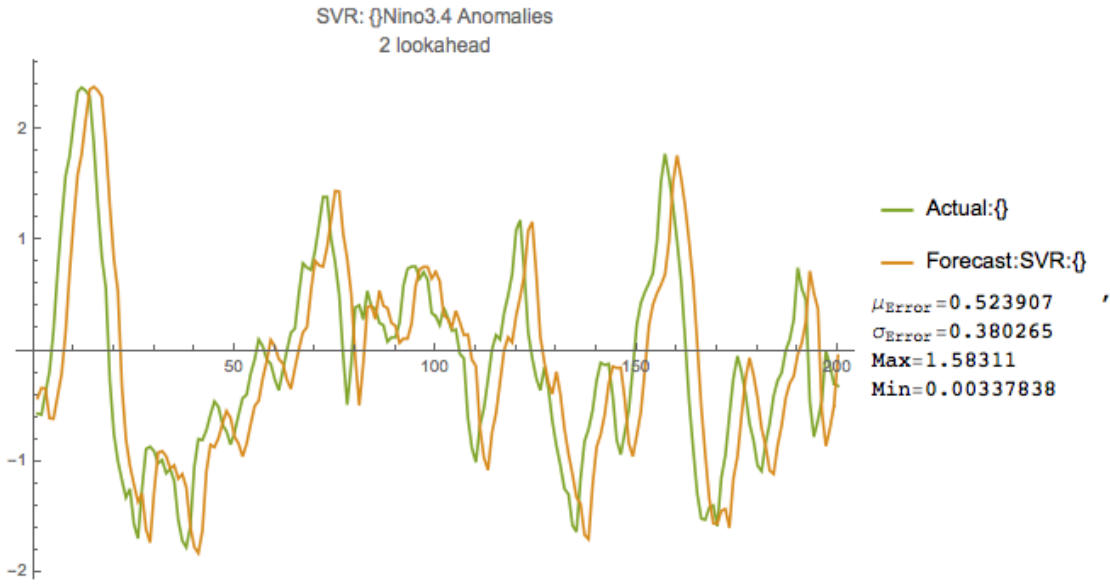
Cat-mice chase SVR trying to catch up with the signal, like unto a person walking on a tight rope has to constantly wobble in order to stay on, the SVR wobbles around the original signal.



2-Month Ahead Lookup

Average Error 21% .

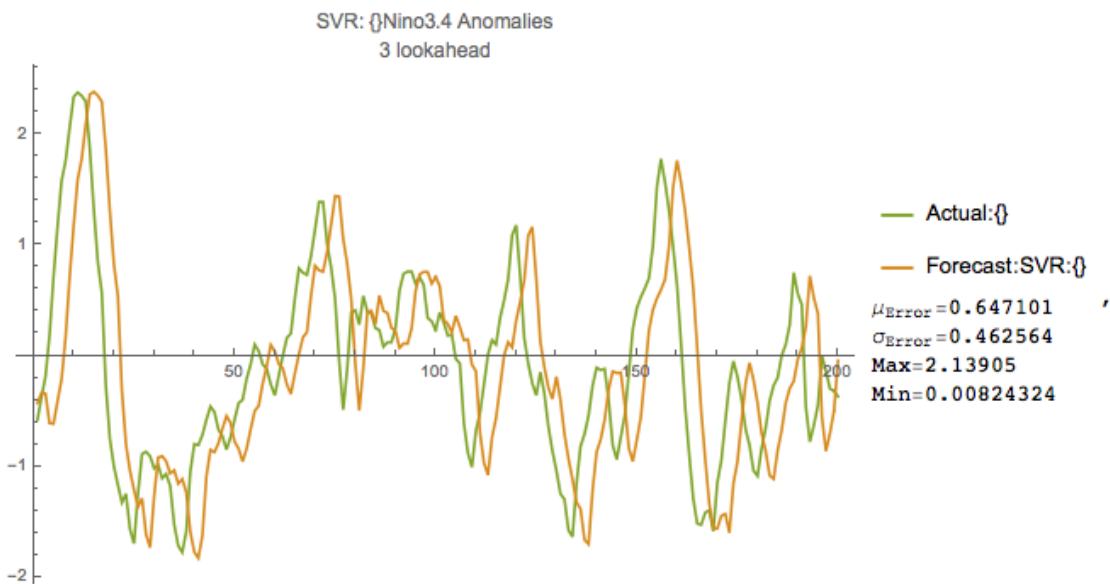
At Max 1.5 unacceptable for forecast.



3-Month Ahead Lookup

Average Error 27% .

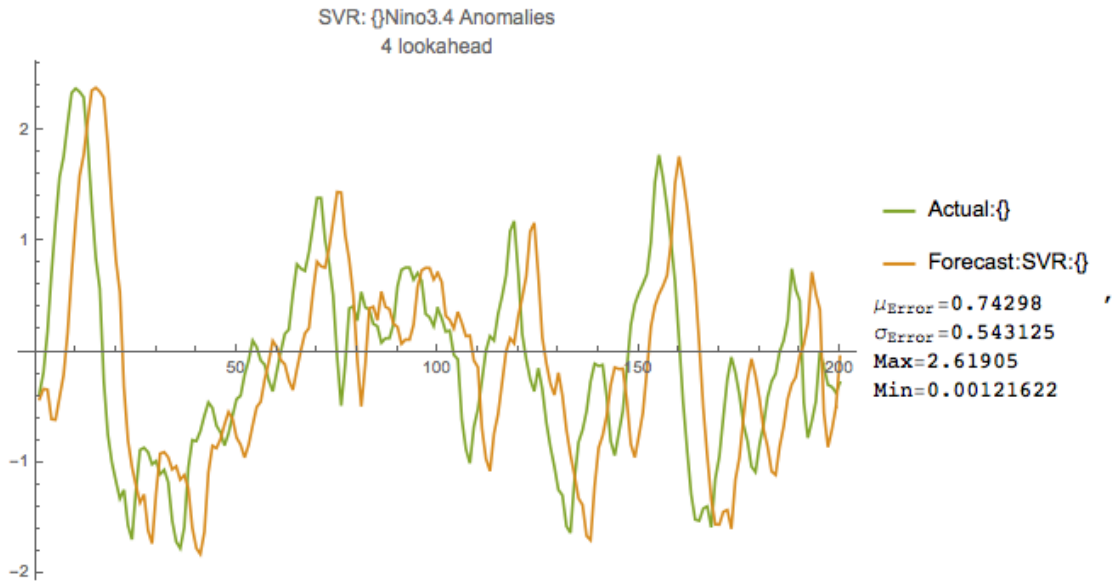
At Max 2.1 unacceptable for forecast.



4-Month Ahead Lookup

Average Error 31% .

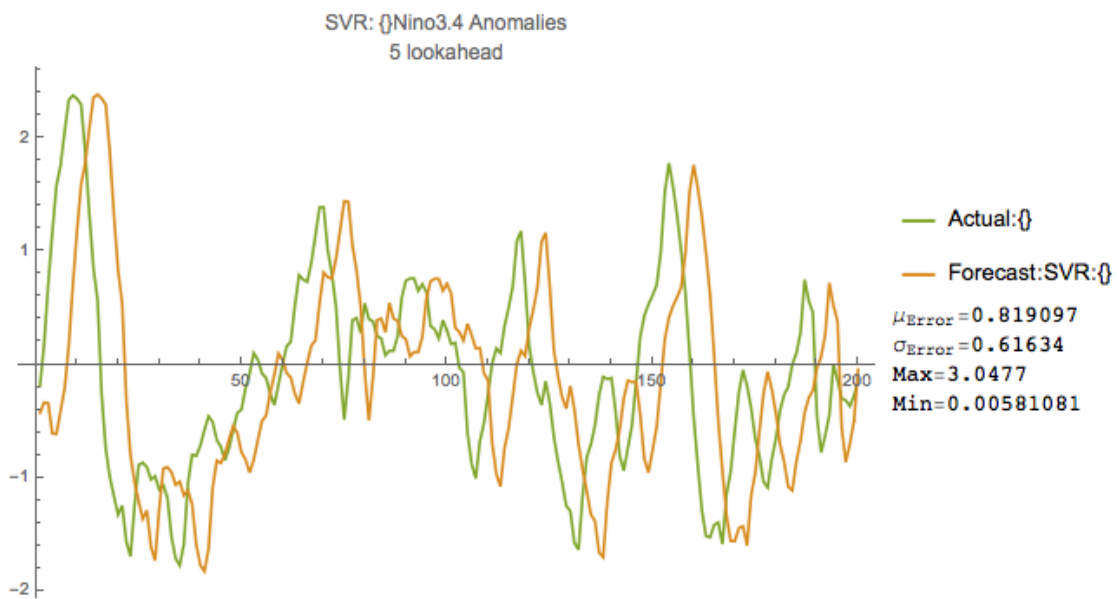
At Max 2.6 unacceptable for forecast.



5-Month Ahead Lookup

Average Error 34% .

At Max 3.0 unacceptable for forecast.



Forecast Configuration

Write up by B. Paláncz

ϵ - Insensitive loss function

The problem of regression is that of finding a function which approximates mapping from an input domain to the real numbers based on a training sample. We refer to the difference between the hypothesis output and its training value as the residual of the output., an indication of the accuracy of the fit at this point. We must decide how to measure the importance of this accuracy, as small residuals may be inevitable while we wish to avoid large ones. The loss function determines this measure. Each choice of loss function will result in a different overall strategy for performing regression. For example least square regression uses the sum of the squares of the residuals.

Although several different approaches are possible, we will provide an analysis for generalization of regression by introducing a threshold test accuracy θ , beyond which we consider a mistake to have been made. We therefore aim to provide a bound on the probability that a randomly drawn test point will have accuracy less than θ . If we access the training set performance using the same θ , we are effectively using the real - valued regressors as classifiers and the worst case lower bounds apply. What we must do in order to make use of dimension free bounds is to allow a margin in the regression accuracy that corresponds to the margin of a classifier. We will use the symbol γ to denote the margin, which measures the amount by which the training and test set accuracy can differ. It should be emphasized that we are therefore using a different loss function during training and testing, where γ measures the discrepancy between the two losses, implying that training point counts as mistake if its accuracy less than $\theta - \gamma$. One way of visualizing this method of assessing performance is to consider a band of size $\pm (\theta - \gamma)$ around the hypothesis function any training points lying outside this band are considered to be training mistakes. Test points count as mistakes only if they lie outside the wider band $\pm \theta$.

The linear ϵ -insensitive loss function $L^\epsilon(x, y, f)$ is defined by

$$L^\epsilon(x, y, f) = (|y - f(x)|)_\epsilon = \max(0, |y - f(x)| - \epsilon)$$

where f is a real - valued function on a domain X , $x \in X$ and $y \in \mathcal{R}$. Similarly the quadratic ϵ -insensitive loss is given by

$$L_2^\epsilon(x, y, f) = (|y - f(x)|)_\epsilon^2$$

Support Vector Regression

SVR uses an admissible kernel, which satisfies the Mercer 's condition to map the data in input space to a high-dimensional feature space in which we can process a regression problem in linear form. Let $x \in \mathbb{R}^n$ and $y \in \mathcal{R}$, where \mathbb{R}^n represents input space. By some nonlinear mapping Φ , the vector x is mapped into a feature space in which a linear regressor function is defined,

$$y = f(x, w) = \langle w, \Phi(x) \rangle + b$$

We seek to estimate this f function based on independent uniformly distributed data $\{\{x_1, y_1\}, \dots, \{x_m, y_m\}\}$, by finding w which minimizing the quadratic ϵ - insensitive losses, with $\epsilon = \theta - \gamma$, namely the following function should be minimize [11]

$$c \sum_{i=1}^m L_2^\epsilon(x_i, y_i, f) + \frac{1}{2} (\|w\|)^2 \rightarrow \min$$

where w is weight vector and c is a constant parameter.

Considering dual representation of a linear regressor in (19), $f(x)$ can be expressed as [12],

$$f(x) = \sum_{i=1}^m \beta_i y_i \langle \Phi(x_i), \Phi(x) \rangle + b$$

which means that the regressor can be expressed as a linear combination of the training points. Consequently using an admissible kernel, a kernel satisfying the Mercer' s condition, we get

$$f(x) = \sum_{i=1}^m \beta_i y_i K(x_i, x) + b = \sum_{i=1}^m \alpha_i K(x_i, x) + b$$

By using Lagrange multiplier techniques, the minimization problem of (20) leads to the following dual optimization problem [12],

$$\begin{aligned} \text{maximize } W(\alpha) &= \sum_{i=1}^m y_i \alpha_i - \epsilon \sum_{i=1}^m |\alpha_i| - \frac{1}{2} \sum_{i,j=1}^m \alpha_i \alpha_j \left(K(x_i, x_j) + \frac{1}{c} \delta_{ij} \right) \\ \text{subject to } &\sum_{i=1}^m \alpha_i = 0 \end{aligned}$$

Let

$$f(x) = \sum_{i=1}^m \alpha_i^* K(x_i, x) + b^*$$

where α^* is the solution of the quadratic optimization problem and b^* is chosen so that $f(x_i) = y_i - \epsilon - \frac{\alpha_i^*}{c}$ for any $\alpha_i^* > 0$.

For samples are inside the ϵ - tube, $\{x_j : |f(x_j) - y_j| < \epsilon\}$, the corresponding α_j^* is zero. It means, we do not need these samples to describe the weight vector w . Consequently

$$f(x) = \sum_{i \in \mathcal{SV}} \alpha_i^* K(x_i, x) + b^*$$

where

$$\mathcal{SV} = \{i : |f(x_i) - y_i| \geq \epsilon \text{ OR } \alpha_i^* > 0\}$$

These x_i sample vectors, $\{x_i : i \in \mathcal{SV}\}$, that come with nonvanishing coefficients α_i^* are called support vectors.

The main features of SVR

We summarize briefly the main features of the SVR,

- application of nonlinear mapping of data from input space into a feature space, in which linear regressor (machine or learning machine) is used. The dual representation of linear regressor leads to the

employment of kernel function.

- quadratic ϵ - insensitive loss function is used as objective function with regularization term for estimation of the weight vector in the kernel representation of the regressor function f , ensuring ϵ accuracy, and leading to a part of samples, called support vector, only which influence the weight vector.

Wavelet kernel

Wavelet kernel with $a \in \mathbb{R}^1$ and all compact $X \subset \mathbb{R}^n$,

$$K(x, z) = \prod_{i=1}^n \left(\cos \left[1.75 \frac{x_i - z_i}{a} \right] \exp \left[- \frac{(x_i - z_i)^2}{2 a^2} \right] \right)$$

37 Months of Training Data

$\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}, \alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{18}, \alpha_{19}, \alpha_{20}, \alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}, \alpha_{25}, \alpha_{26}, \alpha_{27}, \alpha_{28}, \alpha_{29}, \alpha_{30}, \alpha_{31}, \alpha_{32}, \alpha_{33}, \alpha_{34}, \alpha_{35}, \alpha_{36}, \alpha_{37}\}$

Parameters

$\epsilon = 0.025$ to snug close fit

$C = 700$

$a = 0.03$ for wavelet kernel

These parameters were found through manual search, but in future a grid-optimizer will compute them.

Quadratic: All Vectors

Out[296]= -0.025

$$\begin{aligned}
& (\text{Abs}[\alpha_1] + \text{Abs}[\alpha_2] + \text{Abs}[\alpha_3] + \text{Abs}[\alpha_4] + \text{Abs}[\alpha_5] + \text{Abs}[\alpha_6] + \text{Abs}[\alpha_7] + \text{Abs}[\alpha_8] + \text{Abs}[\alpha_9] + \\
& \quad \text{Abs}[\alpha_{10}] + \text{Abs}[\alpha_{11}] + \text{Abs}[\alpha_{12}] + \text{Abs}[\alpha_{13}] + \text{Abs}[\alpha_{14}] + \text{Abs}[\alpha_{15}] + \text{Abs}[\alpha_{16}] + \text{Abs}[\alpha_{17}] + \\
& \quad \text{Abs}[\alpha_{18}] + \text{Abs}[\alpha_{19}] + \text{Abs}[\alpha_{20}] + \text{Abs}[\alpha_{21}] + \text{Abs}[\alpha_{22}] + \text{Abs}[\alpha_{23}] + \text{Abs}[\alpha_{24}] + \\
& \quad \text{Abs}[\alpha_{25}] + \text{Abs}[\alpha_{26}] + \text{Abs}[\alpha_{27}] + \text{Abs}[\alpha_{28}] + \text{Abs}[\alpha_{29}] + \text{Abs}[\alpha_{30}] + \text{Abs}[\alpha_{31}] + \\
& \quad \text{Abs}[\alpha_{32}] + \text{Abs}[\alpha_{33}] + \text{Abs}[\alpha_{34}] + \text{Abs}[\alpha_{35}] + \text{Abs}[\alpha_{36}] + \text{Abs}[\alpha_{37}]) - \\
& 0.53 \alpha_1 - 0.56 \alpha_2 - 0.43 \alpha_3 - 0.4 \alpha_4 - 0.23 \alpha_5 - 0.01 \alpha_6 + \\
& 0.11 \alpha_7 + 0.02 \alpha_8 - \\
& 0.19 \alpha_9 + 0.44 \alpha_{10} + 0.2 \alpha_{11} + \\
& 0.36 \alpha_{12} + 0.34 \alpha_{13} + 0.2 \alpha_{14} - 0.14 \alpha_{15} - \\
& 0.2 \alpha_{16} - 0.27 \alpha_{17} - 0.15 \alpha_{18} - 0.23 \alpha_{19} - \\
& 0.05 \alpha_{20} + 0.25 \alpha_{21} + 0.18 \alpha_{22} + 0.25 \alpha_{23} + \\
& 0.14 \alpha_{24} + 0.07 \alpha_{25} + 0.2 \alpha_{26} + 0.1 \alpha_{27} + 0.18 \alpha_{28} + \\
& 0.47 \alpha_{29} - 0.19 \alpha_{30} - 0.1 \alpha_{31} - 0.92 \alpha_{32} - 0.32 \alpha_{33} + \\
& 0.15 \alpha_{34} + 0.18 \alpha_{35} + 0.45 \alpha_{36} - 0.14 \alpha_{37} + \\
& \frac{1}{2} \left(0. - 1.00143 \alpha_1^2 + 2.25502 \times 10^{-242} \alpha_1 \alpha_2 - 1.00143 \alpha_2^2 + 2.25502 \times 10^{-242} \alpha_2 \alpha_3 - \right. \\
& \quad 1.00143 \alpha_3^2 + 2.25502 \times 10^{-242} \alpha_3 \alpha_4 - 1.00143 \alpha_4^2 + 2.25502 \times 10^{-242} \alpha_4 \alpha_5 - 1.00143 \alpha_5^2 + \\
& \quad 2.25502 \times 10^{-242} \alpha_5 \alpha_6 - 1.00143 \alpha_6^2 + 2.25502 \times 10^{-242} \alpha_6 \alpha_7 - 1.00143 \alpha_7^2 + \\
& \quad 2.25502 \times 10^{-242} \alpha_7 \alpha_8 - 1.00143 \alpha_8^2 - 1.00143 \alpha_9^2 + 2.25502 \times 10^{-242} \alpha_9 \alpha_{10} - \\
& \quad 1.00143 \alpha_{10}^2 + 2.25502 \times 10^{-242} \alpha_{10} \alpha_{11} - 1.00143 \alpha_{11}^2 + 2.25502 \times 10^{-242} \alpha_{11} \alpha_{12} - \\
& \quad 1.00143 \alpha_{12}^2 + 2.25502 \times 10^{-242} \alpha_{12} \alpha_{13} + 2.25502 \times 10^{-242} \alpha_{12} \alpha_{13} - 1.00143 \alpha_{13}^2 + \\
& \quad 2.25502 \times 10^{-242} \alpha_2 \alpha_{14} + 2.25502 \times 10^{-242} \alpha_{13} \alpha_{14} - 1.00143 \alpha_{14}^2 + 2.25502 \times 10^{-242} \alpha_3 \alpha_{15} + \\
& \quad 2.25502 \times 10^{-242} \alpha_{14} \alpha_{15} - 1.00143 \alpha_{15}^2 + 2.25502 \times 10^{-242} \alpha_4 \alpha_{16} + 2.25502 \times 10^{-242} \alpha_{15} \alpha_{16} - \\
& \quad 1.00143 \alpha_{16}^2 + 2.25502 \times 10^{-242} \alpha_5 \alpha_{17} + 2.25502 \times 10^{-242} \alpha_{16} \alpha_{17} - 1.00143 \alpha_{17}^2 + \\
& \quad 2.25502 \times 10^{-242} \alpha_6 \alpha_{18} + 2.25502 \times 10^{-242} \alpha_{17} \alpha_{18} - 1.00143 \alpha_{18}^2 + 2.25502 \times 10^{-242} \alpha_7 \alpha_{19} + \\
& \quad 2.25502 \times 10^{-242} \alpha_{18} \alpha_{19} - 1.00143 \alpha_{19}^2 + 2.25502 \times 10^{-242} \alpha_8 \alpha_{20} + 2.25502 \times 10^{-242} \alpha_{19} \alpha_{20} - \\
& \quad 1.00143 \alpha_{20}^2 + 2.25502 \times 10^{-242} \alpha_9 \alpha_{21} - 1.00143 \alpha_{21}^2 + 2.25502 \times 10^{-242} \alpha_{10} \alpha_{22} + \\
& \quad 2.25502 \times 10^{-242} \alpha_{21} \alpha_{22} - 1.00143 \alpha_{22}^2 + 2.25502 \times 10^{-242} \alpha_{11} \alpha_{23} + \\
& \quad 2.25502 \times 10^{-242} \alpha_{22} \alpha_{23} - 1.00143 \alpha_{23}^2 + 2.25502 \times 10^{-242} \alpha_{12} \alpha_{24} + 2.25502 \times 10^{-242} \alpha_{23} \alpha_{24} - \\
& \quad 1.00143 \alpha_{24}^2 + 2.25502 \times 10^{-242} \alpha_{13} \alpha_{25} + 2.25502 \times 10^{-242} \alpha_{24} \alpha_{25} - 1.00143 \alpha_{25}^2 + \\
& \quad 2.25502 \times 10^{-242} \alpha_{14} \alpha_{26} + 2.25502 \times 10^{-242} \alpha_{25} \alpha_{26} - 1.00143 \alpha_{26}^2 + 2.25502 \times 10^{-242} \alpha_{15} \alpha_{27} + \\
& \quad 2.25502 \times 10^{-242} \alpha_{26} \alpha_{27} - 1.00143 \alpha_{27}^2 + 2.25502 \times 10^{-242} \alpha_{16} \alpha_{28} + 2.25502 \times 10^{-242} \alpha_{27} \alpha_{28} - \\
& \quad 1.00143 \alpha_{28}^2 + 2.25502 \times 10^{-242} \alpha_{17} \alpha_{29} + 2.25502 \times 10^{-242} \alpha_{28} \alpha_{29} - \\
& \quad 1.00143 \alpha_{29}^2 + 2.25502 \times 10^{-242} \alpha_{18} \alpha_{30} + 2.25502 \times 10^{-242} \alpha_{29} \alpha_{30} - 1.00143 \alpha_{30}^2 + \\
& \quad 2.25502 \times 10^{-242} \alpha_{19} \alpha_{31} + 2.25502 \times 10^{-242} \alpha_{30} \alpha_{31} - 1.00143 \alpha_{31}^2 + 2.25502 \times 10^{-242} \alpha_{20} \alpha_{32} + \\
& \quad 2.25502 \times 10^{-242} \alpha_{31} \alpha_{32} - 1.00143 \alpha_{32}^2 + 2.25502 \times 10^{-242} \alpha_{21} \alpha_{33} - 1.00143 \alpha_{33}^2 + \\
& \quad 2.25502 \times 10^{-242} \alpha_{22} \alpha_{34} + 2.25502 \times 10^{-242} \alpha_{33} \alpha_{34} - 1.00143 \alpha_{34}^2 + 2.25502 \times 10^{-242} \alpha_{23} \alpha_{35} + \\
& \quad 2.25502 \times 10^{-242} \alpha_{34} \alpha_{35} - 1.00143 \alpha_{35}^2 + 2.25502 \times 10^{-242} \alpha_{24} \alpha_{36} + 2.25502 \times 10^{-242} \alpha_{35} \alpha_{36} - \\
& \quad \left. 1.00143 \alpha_{36}^2 + 2.25502 \times 10^{-242} \alpha_{25} \alpha_{37} + 2.25502 \times 10^{-242} \alpha_{36} \alpha_{37} - 1.00143 \alpha_{37}^2 \right)
\end{aligned}$$

Quadratic: Supprt Vectors

Tiny coefficients removed:

Out[295]= -0.025

$$\begin{aligned}
 & (\text{Abs}[\alpha_1] + \text{Abs}[\alpha_2] + \text{Abs}[\alpha_3] + \text{Abs}[\alpha_4] + \text{Abs}[\alpha_5] + \text{Abs}[\alpha_6] + \text{Abs}[\alpha_7] + \text{Abs}[\alpha_8] + \text{Abs}[\alpha_9] + \\
 & \quad \text{Abs}[\alpha_{10}] + \text{Abs}[\alpha_{11}] + \text{Abs}[\alpha_{12}] + \text{Abs}[\alpha_{13}] + \text{Abs}[\alpha_{14}] + \text{Abs}[\alpha_{15}] + \text{Abs}[\alpha_{16}] + \text{Abs}[\alpha_{17}] + \\
 & \quad \text{Abs}[\alpha_{18}] + \text{Abs}[\alpha_{19}] + \text{Abs}[\alpha_{20}] + \text{Abs}[\alpha_{21}] + \text{Abs}[\alpha_{22}] + \text{Abs}[\alpha_{23}] + \text{Abs}[\alpha_{24}] + \\
 & \quad \text{Abs}[\alpha_{25}] + \text{Abs}[\alpha_{26}] + \text{Abs}[\alpha_{27}] + \text{Abs}[\alpha_{28}] + \text{Abs}[\alpha_{29}] + \text{Abs}[\alpha_{30}] + \text{Abs}[\alpha_{31}] + \\
 & \quad \text{Abs}[\alpha_{32}] + \text{Abs}[\alpha_{33}] + \text{Abs}[\alpha_{34}] + \text{Abs}[\alpha_{35}] + \text{Abs}[\alpha_{36}] + \text{Abs}[\alpha_{37}]) - \\
 & 0.53 \alpha_1 - 0.56 \alpha_2 - 0.43 \alpha_3 - 0.4 \alpha_4 - 0.23 \alpha_5 - 0.01 \alpha_6 + \\
 & 0.11 \alpha_7 + 0.02 \alpha_8 - \\
 & 0.19 \alpha_9 + 0.44 \alpha_{10} + 0.2 \alpha_{11} + \\
 & 0.36 \alpha_{12} + 0.34 \alpha_{13} + 0.2 \alpha_{14} - 0.14 \alpha_{15} - \\
 & 0.2 \alpha_{16} - 0.27 \alpha_{17} - 0.15 \alpha_{18} - 0.23 \alpha_{19} - \\
 & 0.05 \alpha_{20} + 0.25 \alpha_{21} + 0.18 \alpha_{22} + 0.25 \alpha_{23} + \\
 & 0.14 \alpha_{24} + 0.07 \alpha_{25} + 0.2 \alpha_{26} + 0.1 \alpha_{27} + 0.18 \alpha_{28} + \\
 & 0.47 \alpha_{29} - 0.19 \alpha_{30} - 0.1 \alpha_{31} - 0.92 \alpha_{32} - 0.32 \alpha_{33} + \\
 & 0.15 \alpha_{34} + 0.18 \alpha_{35} + 0.45 \alpha_{36} - 0.14 \alpha_{37} + \\
 & \frac{1}{2} (-1.00143 \alpha_1^2 - 1.00143 \alpha_2^2 - 1.00143 \alpha_3^2 - 1.00143 \alpha_4^2 - 1.00143 \alpha_5^2 - 1.00143 \alpha_6^2 - 1.00143 \alpha_7^2 - \\
 & \quad 1.00143 \alpha_8^2 - 1.00143 \alpha_9^2 - 1.00143 \alpha_{10}^2 - 1.00143 \alpha_{11}^2 - 1.00143 \alpha_{12}^2 - 1.00143 \alpha_{13}^2 - \\
 & \quad 1.00143 \alpha_{14}^2 - 1.00143 \alpha_{15}^2 - 1.00143 \alpha_{16}^2 - 1.00143 \alpha_{17}^2 - 1.00143 \alpha_{18}^2 - 1.00143 \alpha_{19}^2 - \\
 & \quad 1.00143 \alpha_{20}^2 - 1.00143 \alpha_{21}^2 - 1.00143 \alpha_{22}^2 - 1.00143 \alpha_{23}^2 - 1.00143 \alpha_{24}^2 - 1.00143 \alpha_{25}^2 - \\
 & \quad 1.00143 \alpha_{26}^2 - 1.00143 \alpha_{27}^2 - 1.00143 \alpha_{28}^2 - 1.00143 \alpha_{29}^2 - 1.00143 \alpha_{30}^2 - 1.00143 \alpha_{31}^2 - \\
 & \quad 1.00143 \alpha_{32}^2 - 1.00143 \alpha_{33}^2 - 1.00143 \alpha_{34}^2 - 1.00143 \alpha_{35}^2 - 1.00143 \alpha_{36}^2 - 1.00143 \alpha_{37}^2)
 \end{aligned}$$

Out[298]= $\{\alpha_1 \rightarrow -0.481787, \alpha_2 \rightarrow -0.511768, \alpha_3 \rightarrow -0.381961, \alpha_4 \rightarrow -0.352001, \alpha_5 \rightarrow -0.182241,$
 $\alpha_6 \rightarrow -3.15395 \times 10^{-9}, \alpha_7 \rightarrow 0.107345, \alpha_8 \rightarrow 0.0174796, \alpha_9 \rightarrow -0.14229,$
 $\alpha_{10} \rightarrow 0.436876, \alpha_{11} \rightarrow 0.197217, \alpha_{12} \rightarrow 0.356988, \alpha_{13} \rightarrow 0.33702, \alpha_{14} \rightarrow 0.197216,$
 $\alpha_{15} \rightarrow -0.0923643, \alpha_{16} \rightarrow -0.152275, \alpha_{17} \rightarrow -0.222187, \alpha_{18} \rightarrow -0.102353,$
 $\alpha_{19} \rightarrow -0.182237, \alpha_{20} \rightarrow -0.00250159, \alpha_{21} \rightarrow 0.247139, \alpha_{22} \rightarrow 0.177248,$
 $\alpha_{23} \rightarrow 0.24715, \alpha_{24} \rightarrow 0.1373, \alpha_{25} \rightarrow 0.06741, \alpha_{26} \rightarrow 0.197223, \alpha_{27} \rightarrow 0.0973589,$
 $\alpha_{28} \rightarrow 0.177238, \alpha_{29} \rightarrow 0.46683, \alpha_{30} \rightarrow -0.142304, \alpha_{31} \rightarrow -0.0524251, \alpha_{32} \rightarrow -0.871257,$
 $\alpha_{33} \rightarrow -0.272112, \alpha_{34} \rightarrow 0.147289, \alpha_{35} \rightarrow 0.177245, \alpha_{36} \rightarrow 0.446863, \alpha_{37} \rightarrow -0.0923702\}$

SVR Constraints

Out[300]= $-700. < \alpha_1 \leq 700. \&\& -700. < \alpha_2 \leq 700. \&\& -700. < \alpha_3 \leq 700. \&\&$
 $-700. < \alpha_4 \leq 700. \&\& -700. < \alpha_5 \leq 700. \&\& -700. < \alpha_6 \leq 700. \&\& -700. < \alpha_7 \leq 700. \&\&$
 $-700. < \alpha_8 \leq 700. \&\& -700. < \alpha_9 \leq 700. \&\& -700. < \alpha_{10} \leq 700. \&\& -700. < \alpha_{11} \leq 700. \&\&$
 $-700. < \alpha_{12} \leq 700. \&\& -700. < \alpha_{13} \leq 700. \&\& -700. < \alpha_{14} \leq 700. \&\&$
 $-700. < \alpha_{15} \leq 700. \&\& -700. < \alpha_{16} \leq 700. \&\& -700. < \alpha_{17} \leq 700. \&\&$
 $-700. < \alpha_{18} \leq 700. \&\& -700. < \alpha_{19} \leq 700. \&\& -700. < \alpha_{20} \leq 700. \&\& -700. < \alpha_{21} \leq 700. \&\&$
 $-700. < \alpha_{22} \leq 700. \&\& -700. < \alpha_{23} \leq 700. \&\& -700. < \alpha_{24} \leq 700. \&\& -700. < \alpha_{25} \leq 700. \&\&$
 $-700. < \alpha_{26} \leq 700. \&\& -700. < \alpha_{27} \leq 700. \&\& -700. < \alpha_{28} \leq 700. \&\& -700. < \alpha_{29} \leq 700. \&\&$
 $-700. < \alpha_{30} \leq 700. \&\& -700. < \alpha_{31} \leq 700. \&\& -700. < \alpha_{32} \leq 700. \&\& -700. < \alpha_{33} \leq 700. \&\&$
 $-700. < \alpha_{34} \leq 700. \&\& -700. < \alpha_{35} \leq 700. \&\& -700. < \alpha_{36} \leq 700. \&\& -700. < \alpha_{37} \leq 700. \&\&$
 $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10} + \alpha_{11} + \alpha_{12} + \alpha_{13} + \alpha_{14} + \alpha_{15} + \alpha_{16} + \alpha_{17} + \alpha_{18} + \alpha_{19} + \alpha_{20} +$
 $\alpha_{21} + \alpha_{22} + \alpha_{23} + \alpha_{24} + \alpha_{25} + \alpha_{26} + \alpha_{27} + \alpha_{28} + \alpha_{29} + \alpha_{30} + \alpha_{31} + \alpha_{32} + \alpha_{33} + \alpha_{34} + \alpha_{35} + \alpha_{36} + \alpha_{37} == 0$

Sample Quadratic for June 2013

64 corresponds to June 2013.

```

Out[303]= -0.0458108 -
0.14229 e-555.556 (62-x1)2-555.556 (1-x2)2 Cos [58.3333 (62 - x1)] Cos [58.3333 (1 - x2)] +
0.247139 e-555.556 (63-x1)2-555.556 (1-x2)2 Cos [58.3333 (63 - x1)] Cos [58.3333 (1 - x2)] -
0.272112 e-555.556 (64-x1)2-555.556 (1-x2)2 Cos [58.3333 (64 - x1)] Cos [58.3333 (1 - x2)] +
0.436876 e-555.556 (62-x1)2-555.556 (2-x2)2 Cos [58.3333 (62 - x1)] Cos [58.3333 (2 - x2)] +
0.177248 e-555.556 (63-x1)2-555.556 (2-x2)2 Cos [58.3333 (63 - x1)] Cos [58.3333 (2 - x2)] +
0.147289 e-555.556 (64-x1)2-555.556 (2-x2)2 Cos [58.3333 (64 - x1)] Cos [58.3333 (2 - x2)] +
0.197217 e-555.556 (62-x1)2-555.556 (3-x2)2 Cos [58.3333 (62 - x1)] Cos [58.3333 (3 - x2)] +
0.24715 e-555.556 (63-x1)2-555.556 (3-x2)2 Cos [58.3333 (63 - x1)] Cos [58.3333 (3 - x2)] +
0.177245 e-555.556 (64-x1)2-555.556 (3-x2)2 Cos [58.3333 (64 - x1)] Cos [58.3333 (3 - x2)] +
0.356988 e-555.556 (62-x1)2-555.556 (4-x2)2 Cos [58.3333 (62 - x1)] Cos [58.3333 (4 - x2)] +
0.1373 e-555.556 (63-x1)2-555.556 (4-x2)2 Cos [58.3333 (63 - x1)] Cos [58.3333 (4 - x2)] +
0.446863 e-555.556 (64-x1)2-555.556 (4-x2)2 Cos [58.3333 (64 - x1)] Cos [58.3333 (4 - x2)] -
0.481787 e-555.556 (61-x1)2-555.556 (5-x2)2 Cos [58.3333 (61 - x1)] Cos [58.3333 (5 - x2)] +
0.33702 e-555.556 (62-x1)2-555.556 (5-x2)2 Cos [58.3333 (62 - x1)] Cos [58.3333 (5 - x2)] +
0.06741 e-555.556 (63-x1)2-555.556 (5-x2)2 Cos [58.3333 (63 - x1)] Cos [58.3333 (5 - x2)] -
0.0923702 e-555.556 (64-x1)2-555.556 (5-x2)2 Cos [58.3333 (64 - x1)] Cos [58.3333 (5 - x2)] -
0.511768 e-555.556 (61-x1)2-555.556 (6-x2)2 Cos [58.3333 (61 - x1)] Cos [58.3333 (6 - x2)] +
0.197216 e-555.556 (62-x1)2-555.556 (6-x2)2 Cos [58.3333 (62 - x1)] Cos [58.3333 (6 - x2)] +
0.197223 e-555.556 (63-x1)2-555.556 (6-x2)2 Cos [58.3333 (63 - x1)] Cos [58.3333 (6 - x2)] -
0.381961 e-555.556 (61-x1)2-555.556 (7-x2)2 Cos [58.3333 (61 - x1)] Cos [58.3333 (7 - x2)] -
0.0923643 e-555.556 (62-x1)2-555.556 (7-x2)2 Cos [58.3333 (62 - x1)] Cos [58.3333 (7 - x2)] +
0.0973589 e-555.556 (63-x1)2-555.556 (7-x2)2 Cos [58.3333 (63 - x1)] Cos [58.3333 (7 - x2)] -
0.352001 e-555.556 (61-x1)2-555.556 (8-x2)2 Cos [58.3333 (61 - x1)] Cos [58.3333 (8 - x2)] -
0.152275 e-555.556 (62-x1)2-555.556 (8-x2)2 Cos [58.3333 (62 - x1)] Cos [58.3333 (8 - x2)] +
0.177238 e-555.556 (63-x1)2-555.556 (8-x2)2 Cos [58.3333 (63 - x1)] Cos [58.3333 (8 - x2)] -
0.182241 e-555.556 (61-x1)2-555.556 (9-x2)2 Cos [58.3333 (61 - x1)] Cos [58.3333 (9 - x2)] -
0.222187 e-555.556 (62-x1)2-555.556 (9-x2)2 Cos [58.3333 (62 - x1)] Cos [58.3333 (9 - x2)] +
0.46683 e-555.556 (63-x1)2-555.556 (9-x2)2 Cos [58.3333 (63 - x1)] Cos [58.3333 (9 - x2)] -
3.15395 × 10-9 e-555.556 (61-x1)2-555.556 (10-x2)2 Cos [58.3333 (61 - x1)] Cos [58.3333 (10 - x2)] -
0.102353 e-555.556 (62-x1)2-555.556 (10-x2)2 Cos [58.3333 (62 - x1)] Cos [58.3333 (10 - x2)] -
0.142304 e-555.556 (63-x1)2-555.556 (10-x2)2 Cos [58.3333 (63 - x1)] Cos [58.3333 (10 - x2)] +
0.107345 e-555.556 (61-x1)2-555.556 (11-x2)2 Cos [58.3333 (61 - x1)] Cos [58.3333 (11 - x2)] -
0.182237 e-555.556 (62-x1)2-555.556 (11-x2)2 Cos [58.3333 (62 - x1)] Cos [58.3333 (11 - x2)] -
0.0524251 e-555.556 (63-x1)2-555.556 (11-x2)2 Cos [58.3333 (63 - x1)] Cos [58.3333 (11 - x2)] +
0.0174796 e-555.556 (61-x1)2-555.556 (12-x2)2 Cos [58.3333 (61 - x1)] Cos [58.3333 (12 - x2)] -
0.00250159 e-555.556 (62-x1)2-555.556 (12-x2)2 Cos [58.3333 (62 - x1)] Cos [58.3333 (12 - x2)] -
0.871257 e-555.556 (63-x1)2-555.556 (12-x2)2 Cos [58.3333 (63 - x1)] Cos [58.3333 (12 - x2)]

```

Sample Wavelet Kernel for June 2013

