Emergence of Integers in Vacuum

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Abstract

A topological ‘process’ for appearance of integers within a vacuum is described. The process allows for explicit global construction of integers, in every locality of the space. Vacuum constantly refreshes the integers and thus at every glance they are de novo constructions. Integers are byproducts of identifications or gluings of points in a topology. Groupoids are used as a ‘structure’ to explicitly construct and represent the integers. The said gluings also introduce non-differentiability, therefore the ever-changing vacuum foam hosts the integers. Each gluing is considered as a form of memory, storing a particular construction of integers.

Keywords: Emergence, Integers, Vacuum, Topology, Groupoids, Cartan Alternating Multi-linear Maps, Banach Space, Identification, Gluing.

Prelude

A highly social species of ant can communicate information about numbers to colony members and also perform simple arithmetic operations. Researchers surveyed a variety of experimental paradigms for studying animal abilities to count, to understand numerical information and to perform simple arithmetic operations. This new research is reported in an article by Dr. Zhanna Rezhikva and Dr. Boris Ryabko to be published in Volume 148, Number 4, pp. 405-434 of the journal Behaviour. [3]

A quantum mechanical system or particle that is bound -- that is, confined spatially—can only take on certain discrete values of energy. This contrasts with classical particles, which can have any energy. These discrete values are called energy levels. The term is commonly used for the energy levels of electrons in atoms or molecules, which are bound by the electric field of the nucleus, but can also refer to energy levels of nuclei or vibrational or rotational energy levels in molecules. [4]

The author had berries for breakfast grown in his backyard, oddly the berries were integer many i.e. could not find $\sqrt{6}$ berries!

We take integers for granted, as the people took for granted the gravity during the life of Newton, as if they i.e. the integers are just there dangling in nowhere and yet they appear everywhere: from ants to plants to human mind to particles. We just believe and find them ‘just’ there, without much thought as how they got into our spaces, to our minds, to other spaces, and other minds.

And evermore odd is the fact that our integers and the ant’s integers and the berries integers and the particle’s integers are all the same! We do not find a slight alteration in our integers e.g. number of our fingers vs. the number of berries.

1. Groupoids as Structure

Let’s use a topological space τ with trivial Groupoid $\mathbf{0}$. Also use Cartan’s construction for differentials: Choose two Banach spaces E and F and define the space of all p-alternating continuous multi-linear mappings from E^p to F as $\mathcal{A}_p(E;F)$. $\mathcal{A}_p(E;F)$ is a closed subspace of $\mathcal{L}_p(E;F)$ the space of continuous multi-linear mappings, both a vector space and the former subspace of the latter. [2]

Remark 1.1: $\mathcal{L}_1(E;F) = \mathcal{A}_1(E;F)$ *for* $p = 1$.

The norm on $\mathcal{L}_p(E;F)$ induces a metric which induces a topology on $\mathcal{L}_p(E;F)$ and a subsequent subspace topology is induced in $\mathcal{A}_p(E;F)$ as the closed subspace of $\mathcal{L}_p(E;F)$. Therefore $\mathcal{A}_p(E;F)$ is endowed with a topology as well.

To each point of the topological space τ we assign a differential i.e. a member of the $\mathcal{A}_p(E;F)$. The latter provides a process or mechanism for the ‘rate of change’ of parameters.

The set of all possible tuples $\left(x \in \tau\,,\ \omega \in \mathcal{A}_p(E;F)\right)$ is the product of the said two topological spaces or $\tau \times \mathcal{A}_p(E;F)$. And the latter product is a topological space in its own right and it is called the Phase-Space or Topological Phase-Space.

The fundamental group of $\tau \times \mathcal{A}_p(E;F)$ is trivial $\mathbf{0}$ group again. Since the linear spaces e.g. vector spaces have trivial Groupoid 0 since all points/vectors can be homotopically reduced/deformed to the $\mathbf{0}$ vector.

Imagine two distinct members of $\tau \times \mathcal{A}_p(E;F)$ i.e. (x, ω) and (x, ω') where $\omega \neq \omega'$. Identify (glue) these two points in the Topological Phase-Space:

$$(x, \omega) \sim (x, \omega') \quad (1.1)$$

In effect the identification glues two different differentials to the same point or, in other words, the identification ‘stores’ the non-differentiability at the point x.

Remark 1.2: *Nondifferentiability requires a form of memory, both short-term and long-term. Therefore nowheredifferentiable spaces are actually memory systems!*

Introduction of the identification produces a loop within the new space. This loop causes the fundamental group to be non-trivial and indeed have a subgroup isomorphic to $\mathbb{Z}$.

Repeat the said process for any cardinality of points in the space and we get partially/entirely nowheredifferentiable space of some kind with 'weird' fundamental group containing as subgroup $\prod_\alpha \mathbb{Z}_\alpha$ where $\mathbb{Z}_\alpha \cong \mathbb{Z}$. Call this space $V = \left(\tau \times \mathcal{A}_p(E;F),\ \sim\right)$.

There is a projection that 'forgets' and thus gives the original topological space τ:

$$\pi : V \longrightarrow \tau \quad (1.2)$$

Therefore the original topological space is the projection of an infinitely complex nowheredifferentiable space and $(\pi,\ V)$ is called Vacuum:

Definition 1.1: Let ~ be a gluing on product space $\tau \times \mathcal{A}_p(E;F)$, the following is called Vacuum

$$\pi : \left(\tau \times \mathcal{A}_p(E;F),\ \sim\right) \longrightarrow \tau \quad (1.3)$$

$$\pi(x,\ \omega) = x \quad x \in \tau,\ \omega \in \mathcal{A}_p(E;F) \quad (1.4)$$

Obviously the Groupoid of the Vacuum has integers as subgroup of its fundamental group. We say the Structure of the space is this Groupoid, and by Structure we mean what makes the space similar to another space [1]. And we further say that Structure of space includes Integers. You might think of the Groupoids as scaffolds holding the space and the integers.

There is no way any bookkeeping possible for the identification ~ i.e. every time we look at the space (observation) different points are RANDOMLY glued to each other!

This randomization concept is to be added to the space Structure i.e. each observation of the space observes a de novo nascent space. Never the same identifications!

Remark 1.3: *The author proposes that topological spaces to be considered as a byproduct of the said projection, not as standalone concepts. Topological spaces are therefore projections of a nowheredifferentiable space (Vacuum) with Groupoid that has Integers as subgroup of its fundamental groups!*

Example 1.1

Let $\tau = \mathbb{R}^2$and $E = \mathbb{R}$ and $F = \mathbb{R}$ therfore $\mathcal{L}_p(E;F)$ for p = 1 are all the linear functions:

$\omega : \mathbb{R} \longrightarrow \mathbb{R}$
$\omega(x) = c\,x$ for some constant $c \in \mathbb{R}$, actually c is 1×1 matrix or 1 dimensional vector of one element

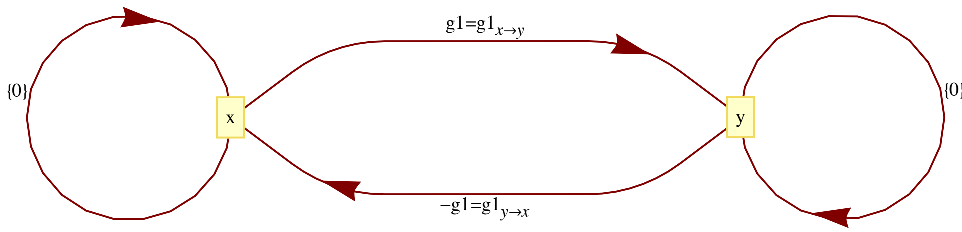
If we represent $\mathcal{A}_1(E;F) = \mathcal{L}_1(E;F)$ by the set of all such constants c i.e. $\mathbb{R}$ then

$$\tau \times \mathcal{A}_p(E;F) = \mathbb{R}^2 \times \mathcal{A}_1(E;F) \cong \mathbb{R}^2 \times \mathbb{R} \cong \mathbb{R}^3$$

Then the trivial Groupoid for $\tau \times \mathcal{A}_p(E; F) = \mathbb{R}^3$ has the following finite graph representation:

FIG 1.1

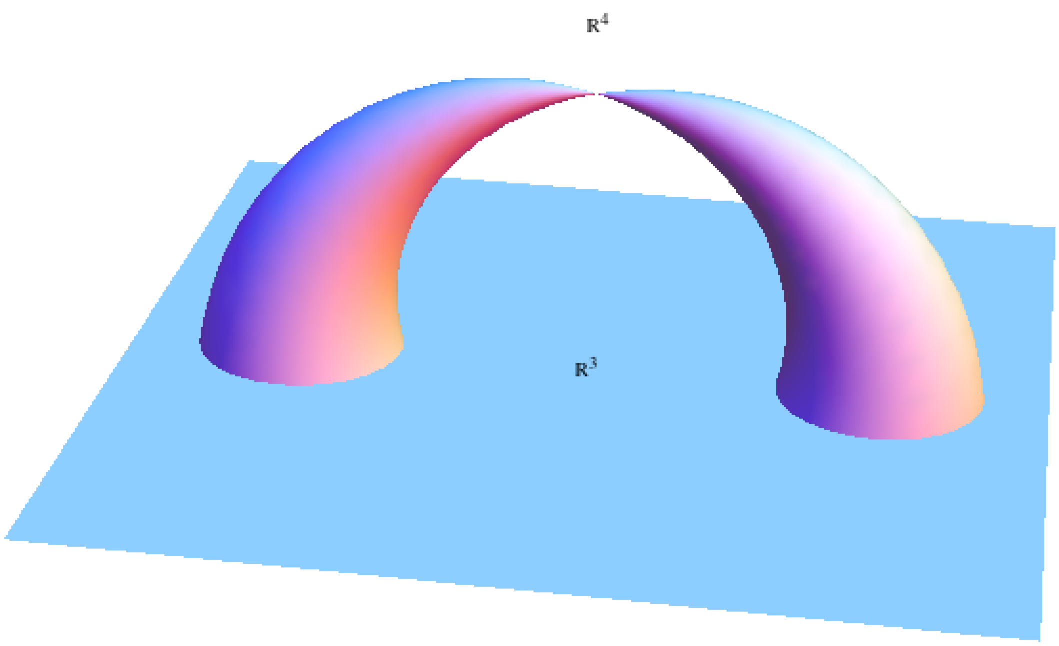
$\mathbb{R}^3$ Groupoid



Imagine $\mathbb{R}^3$ as the ‘gooey’ blue area below and ‘pinch’ two points and pull them and glue them together:

FIG 1.2

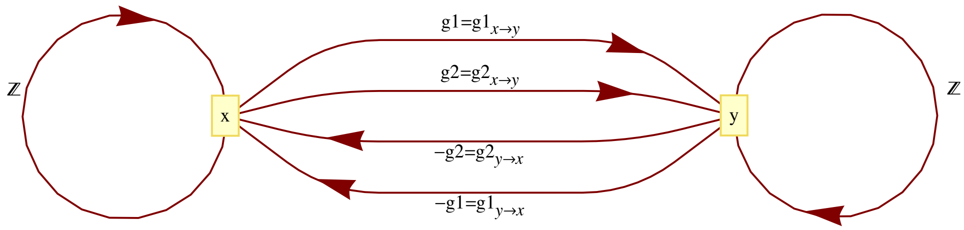
$\tau \times \mathcal{A}_p(E; F) = \mathbb{R}^3$ now has changed it shape (topology)



Therefore the Groupoid for the new topological space after the gluing two points is no longer trivial:

FIG 1.3

$\mathbb{R}^3$ with Two Point Identification



And as it can easily be seen the fundamental group is $\mathbb{Z}$.

Here we say:

The space/vacuum has explicitly constructed the integers

Now let's introduce the non-differentiability via the identification or gluing by (1.1):

$(x, \omega) \sim (x, \omega')$

Where

$\omega : \mathbb{R} \longrightarrow \mathbb{R}$

$\omega(t) = 0.05\, t$

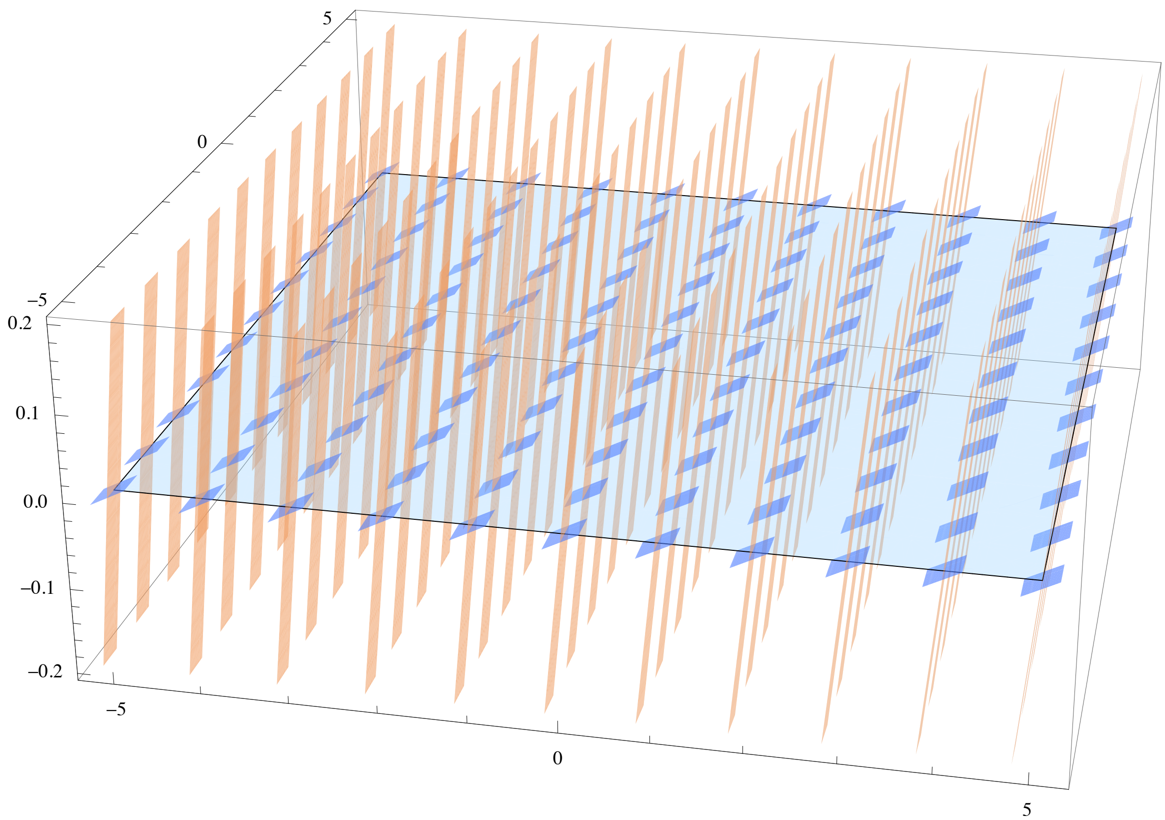
$\omega' : \mathbb{R} \longrightarrow \mathbb{R}$

$\omega'(t) = 1.0\, t$

Keeping t very near to 0, identification (1.1) allows for two differentials at t = 0 which is the point x, this in turn models a non-differentiable construction where the differential is multi-valued i.e. the derivatives do not have a limit.

The following attempts to pictorially render what happens, the longer orange 3D rectangles are groups of lines representing ω and the dark-blue little rectangles ω', larger light-blue rectangle is $\mathbb{R}^2$. If two points are glued to each other at x, then there will be two differentials one the orange and on the dark-blue, at the same time i.e. the derivatives have no limits.

FIG 1.4



2. Loops in Differential Space

Habitually loops are visualized in the spatial space, but in order to understand the nature of the non-differential universe, the same loops are envisaged in the differential spaces or the velocity/momentum spaces.

For ease of visualization let $\tau = \mathbb{R}$ and $E = \mathbb{R}$ and $F = \mathbb{R}$:

$$\tau \times \mathcal{A}_p(E; F) = \mathbb{R} \times \mathcal{A}_1(E; F) \cong \mathbb{R} \times \mathbb{R} \cong \mathbb{R}^2$$

Now let's introduce the non-differentiability via the identification or gluing by (1.1):

$(x, \omega) \sim (x, \omega')$ both tuples belong to $\mathbb{R}^2$

Where

$$\omega : \mathbb{R} \longrightarrow \mathbb{R} \qquad (2.1)$$

$$\omega(t) = c\, t$$

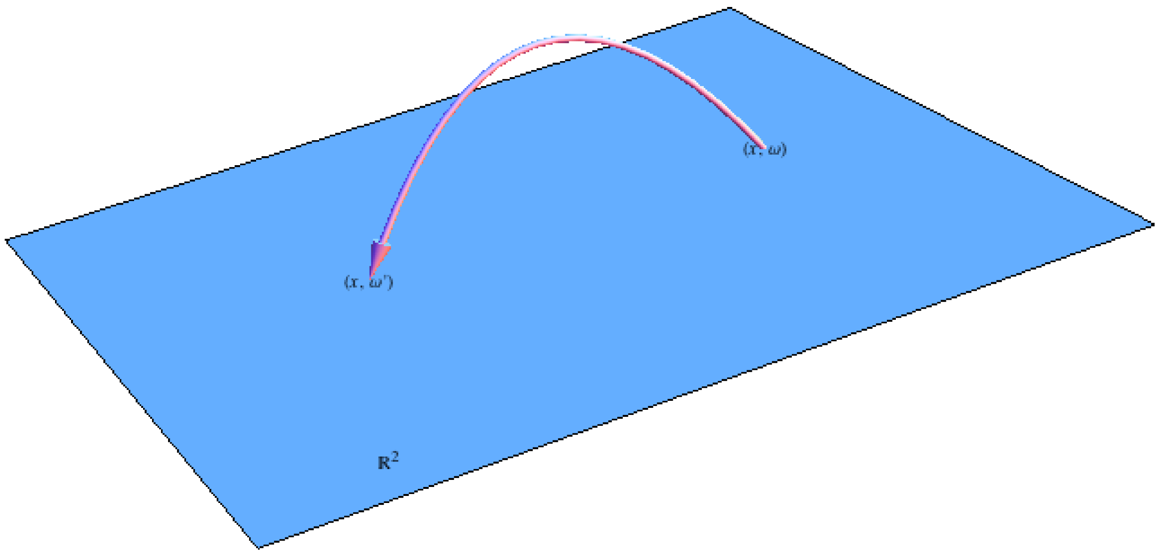
$$\omega' : \mathbb{R} \longrightarrow \mathbb{R}$$

$\omega'(t) = c' t$

And further assume $c \neq c'$.

We visualize this identification as following rendition, where one end of the arrow is (x, ω) and the other endpoint (x, ω') and the arrow has direction as in a loop :

F I G
2.1



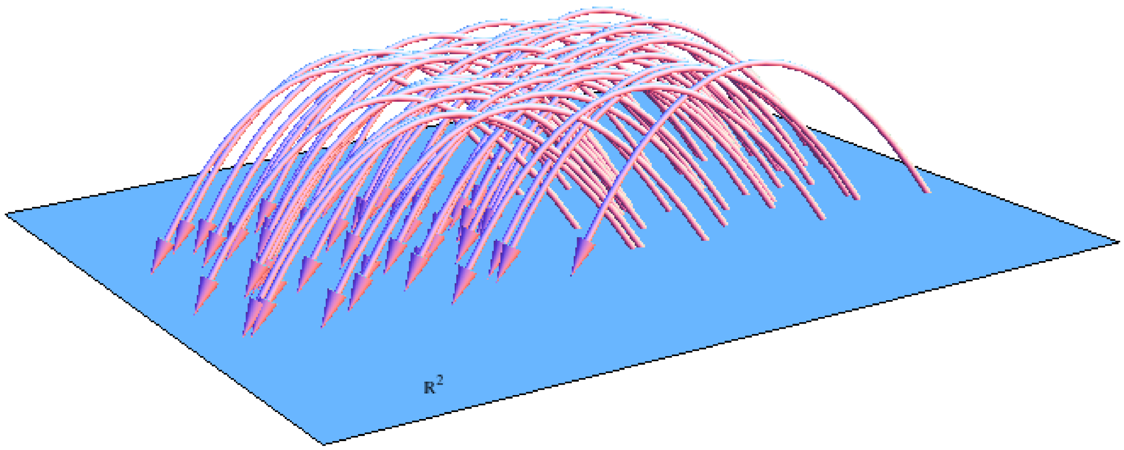
2.1 Explicit Construction of Integers

In FIG 2.1 the looping around the loop in one direction generates the positive integers and looping in reverse generates the negative integers.

No matter where a point is on the Topological space τ a path can connect it to the said loop i.e. the way the integers were generated is global throughout the space, no matter where/when the identification or gluing took place.

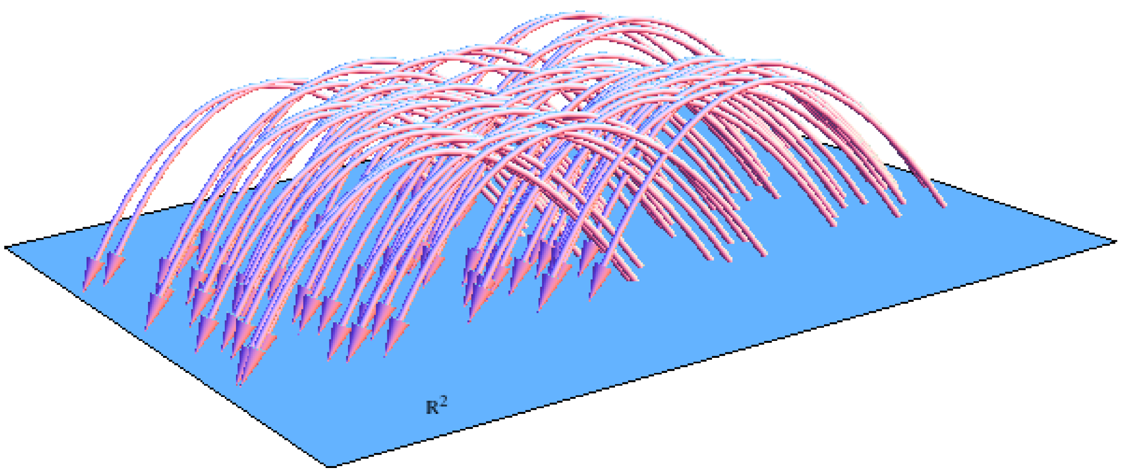
A 1-D Brownian Motion or Random-walk can be rendered topologically as following, each arrow/loop represents a 'jitter' or a broken line sengment along the path of motion:

F I G
2.1.1



Every occasion we look at this Brownian Motion we see a different configuration:

F I G
2.1.2



The latter is the core concept of randomness in the vacuum! And integers are refreshed de novo each time we think of them or compute with them. Thus this notion of vacuum can be rendered as a foamy random ever-changing construction, however with a constant fixed projection (1.2).

Remark 2.1.1: *These loops are the topological generalization of* $e^{i\pi S/\hbar}$ *if you visualize the complex exponentiation geometrically as rapid looping around a circle*.

2.2 Loops as Memory

Expression (1.1) conveys the concept of memory location:

$(x, \omega) \sim (x, \omega')$

At location x, there is an infinitesimal loop, connecting x to x, and the loop is non-differentiable. The loop is not in spatial space but in the derivative or the differential space.

The said loop identifies: At point x a new information or knowledge has been interjected.

Therefore FIG 2.1.2 identifies a huge amount of information interjected into the space, or the Brownian Motion identifies or declares the incoming influx of large amounts of information:

From any entity subject to influx of information/knowledge or subject to learning/forgetting, emerges spatial attributes and counting i.e. space with integers.

To read more about the influx of information and non-differentiablity see:

http://lossofgenerality.devzing.com/blog/2012/06/29/discontinuity-and-flux-of-informationknowledge/

2.3 Emergence

Above a vacuum was described, with infinitely many gluing and and a projection. None of which has anything to do with the integers. But from infinite refreshing of this vacuum EMERGES the integers, ceaselessly, like from the molecules of water in a lake emerges the waves.

There are no waves in the individual molecules of water or small ensembles of theirs, but waves are as real as the molecules of water themselves.

The word emerge was used carefully to cognize a concept of something emerging from the micro world of vacuum into the macro and yet it has no equal or similar in the micro world.

3. Coda

Traditionally, in most primitive way, we separate space from the ant and from the arithmetic of the ant. Indeed a better way of thinking is the more integrated notion:

Ant is not inside the space, and the integers arithmetic is not inside the ant’s mind, the space the ant the integer arithmetic are all manifestations of the same entity.

Ant: its design and concrescence [5] are subject to learning and forgetting, it is subject influx and out-flux information. Ant’s DNA replication and regeneration is learning and its death result of forgetting.

Mutations, Evolutions, Designs: All require certain randomness, and that randomness is linked to the nowhered-

ifferentiable structures, and those as conceptualized earlier in this paper, are the robust process behind explicit construction of integers.

Learning and Forgetting, influx and out-flux of information, nowheredifferentiable structures and integers, are all manifestations of one and the same entity, emerging from which spatial attributes and counting.

Appendix

Homotopy and Groupoids:

http://lossofgenerality.devzing.com/blog/2011/10/04/homotopy/

This is super quick pictorial to motivate the concept of homotopy and grouopids.

Examples of groupoids:

http://lossofgenerality.devzing.com/blog/2011/10/05/circle/
http://lossofgenerality.devzing.com/blog/2011/10/05/perforated-circle/
http://lossofgenerality.devzing.com/blog/2011/10/04/perforated-2d-plane/
http://lossofgenerality.devzing.com/blog/2011/10/04/sphere/
http://lossofgenerality.devzing.com/blog/2011/10/04/perforated-sphere/
http://lossofgenerality.devzing.com/blog/2011/10/05/cylinder/

References

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http://www.mediafire.com/file/8dd8t8csonxyd8a/MATH%20Cartan%20Diff%20Forms%2C%20Chap%201.pdf

[3] Full Text: http://brill.publisher.ingentaconnect.com/content/brill/beh/pre-prints/beh2917

[4] http://en.wikipedia.org/wiki/Energy_level

[5] http://en.wikipedia.org/wiki/Philosophy_of_Organism