Heebie-jeebies Symbolic Computations

In[62]:=
```
<< Notation`
```

In[63]:=
```
Notation[ < T_ > ⟹ Mean[T_] ]
```

$$\text{Notation}\left[\ C_{i\_,j\_}^{(t\_)}(-\tau\_) \Longrightarrow \text{FullSimplify}\left[\frac{\left(\langle T_{i\_}(t\_) * T_{j\_}(t\_-\tau\_)\rangle\right) - \left(\langle T_{i\_}(t\_)\rangle\right) * \left(\langle T_{j\_}(t\_-\tau\_)\rangle\right)}{\sqrt{\left(\left\langle \left(T_{i\_}(t\_) - \left(\langle T_{j\_}(t\_)\rangle\right)\right)^2\right\rangle\right) * \left(\left\langle \left(T_{i\_}(t\_-\tau\_) - \left(\langle T_{j\_}(t\_-\tau\_)\rangle\right)\right)^2\right\rangle\right)}}\right]\right]$$

```
Notation[ T(t_ - τ_) ⟹ Join[Drop[T, τ_], ConstantArray[0, τ_]] ]
Notation[ T_{i_}(t_ - τ_) ⟹ Join[Drop[T[[i_]], τ_], ConstantArray[0, τ_]] ]
Notation[ T_{i_}(t_) ⟹ T[[i_]] ]

id[x_] := x

Notation[ T_{i_} ⟹ T[[i_]] ]
```

Let’s make a simple list (array) of 3-element arrays:

In[70]:=
```
T =.;
T = {{x1, x2, x3}, {y1, y2, y3}, {z1, z2, z3} };
```

Test the index notation:

In[72]:= $\mathbf{T_1\ (t)}$

Out[72]= $\{x1,\ x2,\ x3\}$

In[73]:= $\mathbf{T_1\ (t-1)}$

Out[73]= $\{x2,\ x3,\ 0\}$

In[74]:= $\mathbf{T_1\ (t-2)}$

Out[74]= $\{x3,\ 0,\ 0\}$

In[75]:= $\mathbf{T_3\ (t-1)}$

Out[75]= $\{z2,\ z3,\ 0\}$

In[76]:= $\mathbf{T_3\ (t-2)}$

Out[76]= $\{z3,\ 0,\ 0\}$

In[77]:= $\mathbf{< T_1 >}$

Out[77]= $\frac{1}{3}\ (x1 + x2 + x3)$

In[78]:= $\mathbf{< T_1\ (t-1) >}$

Out[78]= $\frac{x2 + x3}{3}$

In[79]:= $\mathbf{< T_1\ (t) * T_1\ (t-1) >}$

Out[79]= $\frac{1}{3}\ (x1\ x2 + x2\ x3)$

In[80]:= $\mathbf{< T_1\ (t) * T_1\ (t-2) >}$

Out[80]= $\frac{x1\ x3}{3}$

In[83]:= $\mathbf{< T_1\ (t) * T_2\ (t-1) >}$

Out[83]= $\frac{1}{3}\ (x1\ y2 + x2\ y3)$

In[84]:= $\mathbf{< T_1\ (t) * T_3\ (t-2) >}$

Out[84]= $\frac{x1\ z3}{3}$

In[85]:= $\mathbf{< (T_1\ (t) - (< T_2\ (t) >))^2 > //\ FullSimplify}$

Out[85]= $\frac{1}{27}\ \left((-3\ x1 + y1 + y2 + y3)^2 + (-3\ x2 + y1 + y2 + y3)^2 + (-3\ x3 + y1 + y2 + y3)^2\right)$

In[86]:= $\mathbf{< (T_1\ (t-1) - (< T_2\ (t-1) >))^2 > //\ FullSimplify}$

Out[86]= $\frac{1}{9}\ \left(3\ x2^2 + 3\ x3^2 - 2\ x2\ (y2 + y3) - 2\ x3\ (y2 + y3) + (y2 + y3)^2\right)$

In[87]:= $< (T_1\,(t-2) - (< T_2\,(t-2) >))^2 >$ `// FullSimplify`

Out[87]= $$\frac{1}{9}\left(3\,\text{x3}^2 - 2\,\text{x3}\,\text{y3} + \text{y3}^2\right)$$

Let’s test the C notation:

In[88]:= $$\underset{1,2}{\overset{(t)}{C}}(-1)$$

Out[88]= $$-\left(\text{x2}\,(\text{y2} - 2\,\text{y3}) + \text{x1}\,(-2\,\text{y2} + \text{y3}) + \text{x3}\,(\text{y2} + \text{y3})\right) \Big/ \left(\sqrt{\left(\left(3\,\text{x2}^2 + 3\,\text{x3}^2 - 2\,\text{x2}\,(\text{y2} + \text{y3}) - 2\,\text{x3}\,(\text{y2} + \text{y3}) + (\text{y2} + \text{y3})^2\right)\left(3\,\text{x1}^2 + 3\left(\text{x2}^2 + \text{x3}^2\right) - 2\,\text{x1}\,(\text{y1} + \text{y2} + \text{y3}) - (2\,\text{x2} + 2\,\text{x3} - \text{y1} - \text{y2} - \text{y3})\,(\text{y1} + \text{y2} + \text{y3})\right)\right)}\right)$$

In[89]:= $$\underset{1,1}{\overset{(t)}{C}}(-1)$$

Out[89]= $$-\frac{\text{x2}^2 - \text{x2}\,\text{x3} + \text{x3}^2 + \text{x1}\,(-2\,\text{x2} + \text{x3})}{2\sqrt{\left(\text{x2}^2 - \text{x2}\,\text{x3} + \text{x3}^2\right)\left(\text{x1}^2 + \text{x2}^2 - \text{x2}\,\text{x3} + \text{x3}^2 - \text{x1}\,(\text{x2} + \text{x3})\right)}}$$

In[90]:= $$\underset{1,2}{\overset{(t)}{C}}(-2)$$

Out[90]= $$-\left((-2\,\text{x1} + \text{x2} + \text{x3})\,\text{y3}\right) \Big/ \left(\sqrt{\left(\left(3\,\text{x3}^2 - 2\,\text{x3}\,\text{y3} + \text{y3}^2\right)\left(3\,\text{x1}^2 + 3\left(\text{x2}^2 + \text{x3}^2\right) - 2\,\text{x1}\,(\text{y1} + \text{y2} + \text{y3}) - (2\,\text{x2} + 2\,\text{x3} - \text{y1} - \text{y2} - \text{y3})\,(\text{y1} + \text{y2} + \text{y3})\right)\right)}\right)$$

In[91]:= $$\underset{2,2}{\overset{(t)}{C}}(-2)$$

Out[91]= $$-\frac{\text{y3}\,(-2\,\text{y1} + \text{y2} + \text{y3})}{2\sqrt{\text{y3}^2\left(\text{y1}^2 + \text{y2}^2 - \text{y2}\,\text{y3} + \text{y3}^2 - \text{y1}\,(\text{y2} + \text{y3})\right)}}$$

In[93]:= $$\underset{3,2}{\overset{(t)}{C}}(-2)$$

Out[93]= $$\frac{\sqrt{3}\,\text{y3}\,(2\,\text{z1} - \text{z2} - \text{z3})}{\sqrt{\left((\text{y1} + \text{y2} + \text{y3} - 3\,\text{z1})^2 + (\text{y1} + \text{y2} + \text{y3} - 3\,\text{z2})^2 + (\text{y1} + \text{y2} + \text{y3} - 3\,\text{z3})^2\right)\left(\text{y3}^2 - 2\,\text{y3}\,\text{z3} + 3\,\text{z3}^2\right)}}$$

In[94]:= $$\underset{1,3}{\overset{(t)}{C}}(-2)$$

Out[94]= $$-\left((-2\,\text{x1} + \text{x2} + \text{x3})\,\text{z3}\right) \Big/ \left(\sqrt{\left(\left(3\,\text{x3}^2 - 2\,\text{x3}\,\text{z3} + \text{z3}^2\right)\left(3\,\text{x1}^2 + 3\left(\text{x2}^2 + \text{x3}^2\right) - 2\,\text{x1}\,(\text{z1} + \text{z2} + \text{z3}) - (2\,\text{x2} + 2\,\text{x3} - \text{z1} - \text{z2} - \text{z3})\,(\text{z1} + \text{z2} + \text{z3})\right)\right)}\right)$$