Fitting Differential Equations to Darwin Δ Anomalies

Let f be Darwin Δ and from previous computations we guess that it has the following form:

$$f = h(t)\,\mathrm{Cos}(a\,t) \quad \frac{2\pi}{2.3} \leqslant a \leqslant \frac{2\pi}{4.7}$$

See the Scalogram below.

Take derivative from both sides:

$$f' = -a\,h\,\mathrm{Sin}(a\,t) + h'\,\mathrm{Cos}(a\,t)$$

Replace h :

$$f' = -a\left(\frac{f}{\mathrm{Cos}(a\,t)}\right)\mathrm{Sin}\,(a\,t) + h'\,\mathrm{Cos}\,(a\,t)$$

Let’s approximate $h'\,\mathrm{Cos}(a\,t)$ with a constant (first term in the taylor series) assuming alomst constant rate of change d:

$$f' = -a\left(\frac{f}{\mathrm{Cos}(a\,t)}\right)\mathrm{Sin}\,(a\,t) + d$$

This assumption stems from the strategy that Darwin Δ is periodic and behaves like the Sin or Cos function and its derivative has the known similar behaviour.

Bring all terms to one side:

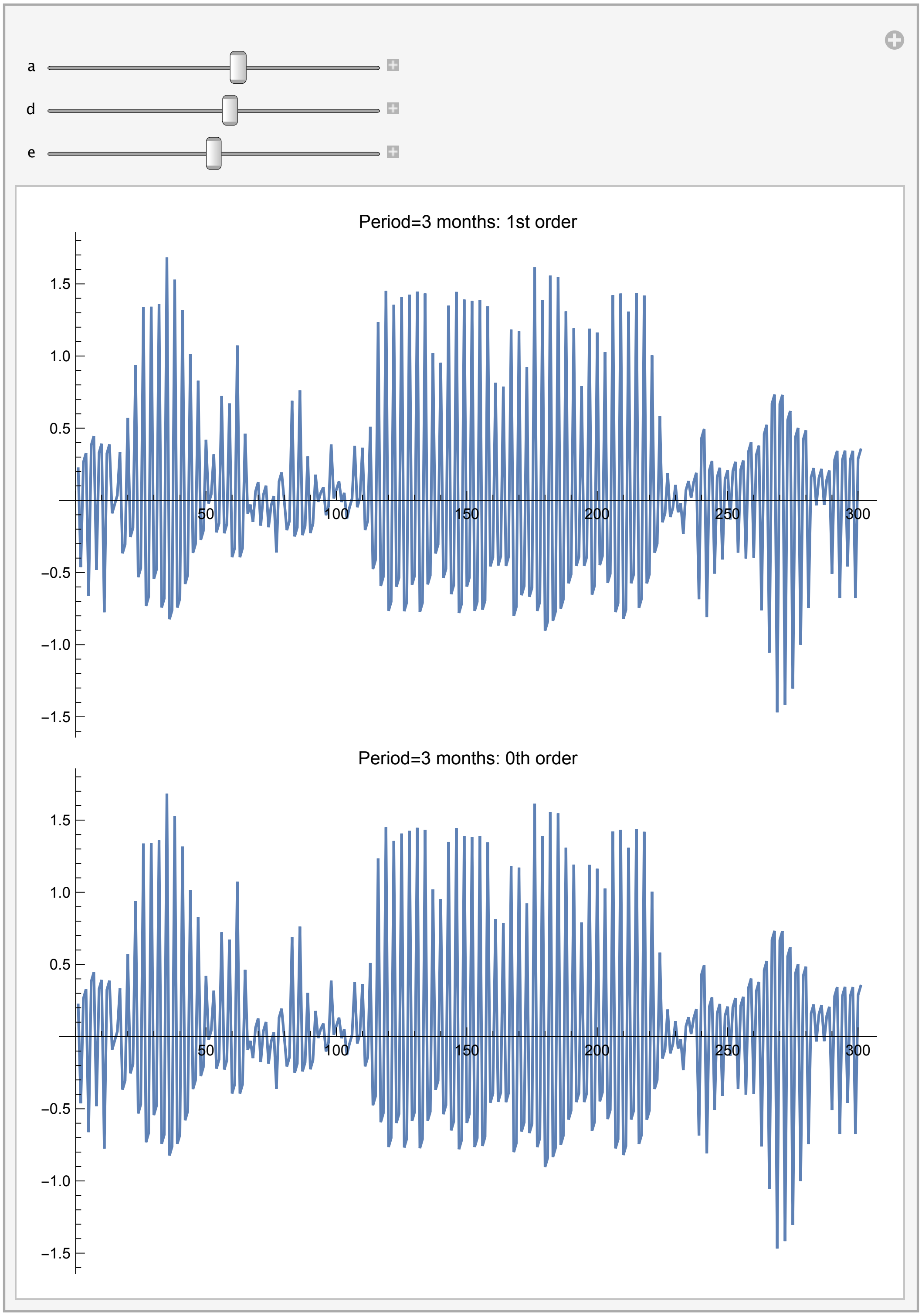
$$f'\,\mathrm{Cos}(a\,t) + a\,f\,\mathrm{Sin}(a\,t) - d\,\mathrm{Cos}(a\,t) = 0$$

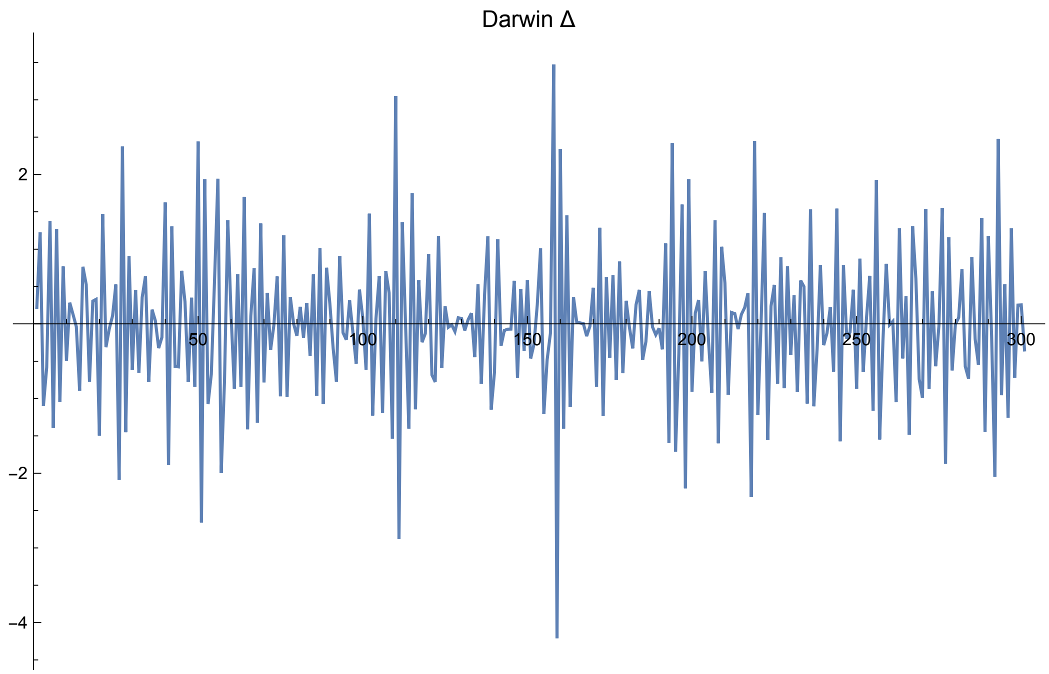
Extend the formulation above to the first order approximation i.e.

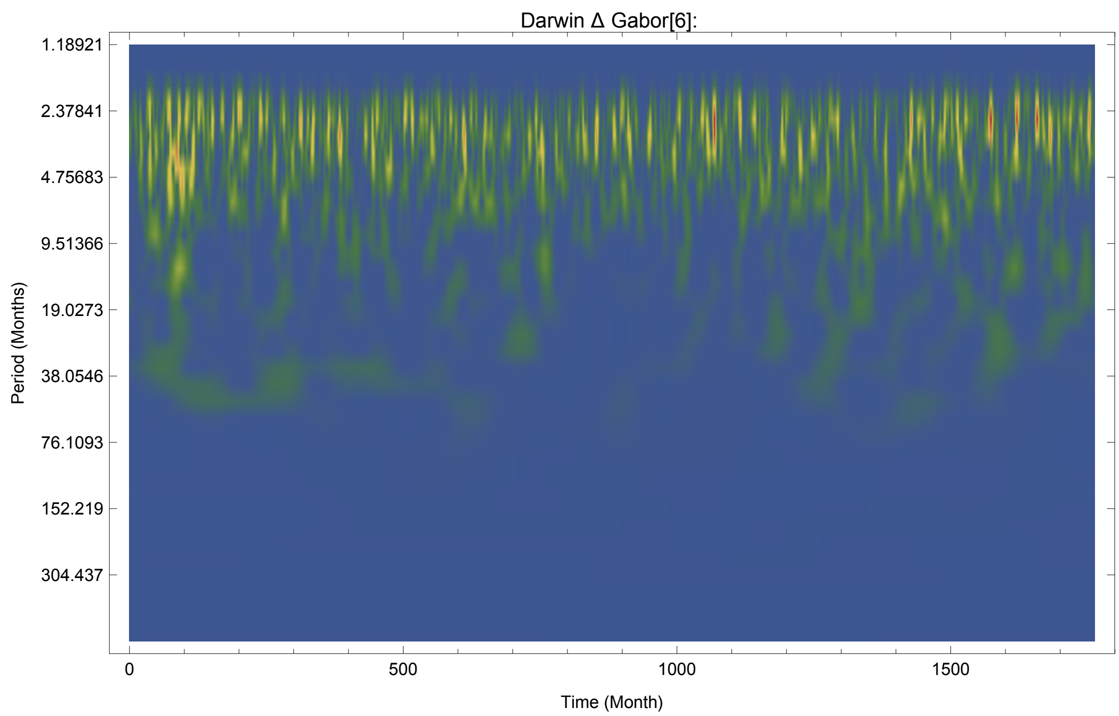
$$f'\,\mathrm{Cos}(a\,t) + a\,f\,\mathrm{Sin}(a\,t) - (d + e\,t)\,\mathrm{Cos}(a\,t) = 0$$

Note that c in the code below was actually picked from the data for Darwin Anomalies, about 300 months prior to the end of 2012 data.

```
Manipulate[
 Clear[f1, f2];
 c = 0.21961;

 s1 = NDSolve[{f1'[t] * Cos[a * t] == -a * f1[t] * Sin[a * t] + (d + e * t) * Cos[a * t],
    f1[1463] == c}, f1, {t, 1463, 1763}, MaxSteps → ∞];

 s2 = NDSolve[{f2'[t] * Cos[a * t] == -a * f2[t] * Sin[a * t] + d * Cos[a * t], f2[1463] == c},
   f2, {t, 1463, 1763}, MaxSteps → ∞];


 Column[{ListLinePlot[Table[Evaluate[f1[t] /. s1][[1]], {t, 1463, 1763, 1}],
    PlotRange → All, PlotLabel →
     "Period=" <> ToString[2 * Pi / a] <> " months: 1st order", ImageSize → 500],
   ListLinePlot[Table[Evaluate[f2[t] /. s2][[1]], {t, 1463, 1763, 1}],
    PlotRange → All, PlotLabel →
     "Period=" <> ToString[2 * Pi / a] <> " months: 0th order", ImageSize → 500]
  }]


 , {a, 2 * Pi / 2, 2 * Pi / 4.7 }, {d, -0.9, 0.9}, {e, -0.000001, 0.000001},
 SaveDefinitions → True, TrackedSymbols → Manipulate, Initialization :→ (a = 2 * Pi / 3;
   e = 0;
   d = 0.1 )]
```


a
d
e
Period=3 months: 1st order
1.5
1.0
0.5
−0.5
−1.0
−1.5
50
100
150
200
250
300
Period=3 months: 0th order
1.5
1.0
0.5
−0.5
−1.0
−1.5
50
100
150
200
250
300


Darwin Δ
2
−2
−4
50
100
150
200
250
300


Darwin Δ Gabor[6]:
1.18921
2.37841
4.75683
9.51366
19.0273
38.0546
76.1093
152.219
304.437
Period (Months)
0
500
1000
1500
Time (Month)