In[1]:=
```
<< Notation`
```

In[2]:=
```
outerFun := Times
Clear[U, H, t, k, α];
Notation[ x_m_,n_ ⟺ x_[[m_]][[n_]] ]
Notation[ | x_ > < y_ | ⟹

  If[StringMatchQ[ToString[FullForm[sites[[y_]]]], "*CirclePlus*"] == True,
   Outer[outerFun, sites[[x_]], Distribute[Conjugate[sites[[y_]]], CirclePlus]],

   Outer[outerFun, sites[[x_]], Conjugate[sites[[y_]]]]]]

Notation[ < x_ | y_ | z_ > ⟹ sites[[x_]].(y_).sites[[z_]] ]
(* the paran around y_ is a must *)
Notation[ ∑^sites expr_ ⟹ Sum[expr_, {m, 1, Length[sites]}, {n, 1, Length[sites]}] ]

Notation[
 t^i_,j_ ⟹ Which[(i_ == 1 && j_ == 2), t, (i_ == 2 && j_ == 1), -t, ( i_ == 2 && j_ == 3), t,
   ( j_ == 3 && i_ == 2), -t, (i_ == 3 && j_ == 1), 2 * t, (j_ == 1 && i_ == 3), -2 * t] ]
Notation[ H^i_,j_ ⟹ k ⅇ^(ⅈ α) | i_ > < j_ | ]
Notation[ U^i_,j_ ⟹ ⅇ^(-ⅈ H^i_,j_ t^i_,j_) ]
```

Sites

In[11]:=
```
sites = {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}};
```

Ket-Bra Outer Product

In[12]:=
```
| 1 > < 2 | // MatrixForm
```

Out[12]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In[13]:=
```
| 2 > < 3 | // MatrixForm
```

Out[13]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

In[14]:=
```
| 3 > < 1 | // MatrixForm
```

Out[14]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$H^{i,j}$$

Needs ‘,’ to spearate indices. Otherwise ambiguities might appear due to typesetting.

In[15]:=
```
H^{1,2} // MatrixForm
```

Out[15]//MatrixForm=

$$\begin{pmatrix} 0 & e^{i\alpha} k & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In[16]:=
```
H^{2,3} // MatrixForm
```

Out[16]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & e^{i\alpha} k \\ 0 & 0 & 0 \end{pmatrix}$$

In[17]:=
```
H^{3,1} // MatrixForm
```

Out[17]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ e^{i\alpha} k & 0 & 0 \end{pmatrix}$$

In[18]:=
```
(H^{1,2} + H^{2,3} + H^{3,1}) // MatrixForm
```

Out[18]//MatrixForm=

$$\begin{pmatrix} 0 & e^{i\alpha} k & 0 \\ 0 & 0 & e^{i\alpha} k \\ e^{i\alpha} k & 0 & 0 \end{pmatrix}$$

$$U^{i,j}$$

In[19]:= $U^{1,2}$ // MatrixForm

Out[19]//MatrixForm=

$$\begin{pmatrix} 1 & e^{-i e^{i\alpha} k t} & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

In[20]:= $U^{2,3}$ // MatrixForm

Out[20]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & e^{-i e^{i\alpha} k t} \\ 1 & 1 & 1 \end{pmatrix}$$

In[21]:= $U^{3,1}$ // MatrixForm

Out[21]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ e^{-2 i e^{i\alpha} k t} & 1 & 1 \end{pmatrix}$$

Trotter Expansion

In[22]:= $U^{1,2}\, U^{2,3}\, U^{3,1}\, U^{2,3}\, U^{1,2}$ // MatrixForm

Out[22]//MatrixForm=

$$\begin{pmatrix} 1 & e^{-2 i e^{i\alpha} k t} & 1 \\ 1 & 1 & e^{-2 i e^{i\alpha} k t} \\ e^{-2 i e^{i\alpha} k t} & 1 & 1 \end{pmatrix}$$

In[23]:=

$U^{1,2}\, U^{2,3}\, U^{3,1}\, U^{3,1}\, U^{2,3}\, U^{1,2}$ // MatrixForm

Out[23]//MatrixForm=

$$\begin{pmatrix} 1 & e^{-2 i e^{i\alpha} k t} & 1 \\ 1 & 1 & e^{-2 i e^{i\alpha} k t} \\ e^{-4 i e^{i\alpha} k t} & 1 & 1 \end{pmatrix}$$

H

In[24]:= $H := k\, e^{i\alpha}$

$((|1\rangle\langle 2|) + (|2\rangle\langle 1|) + (|2\rangle\langle 3|) + (|3\rangle\langle 2|) + (|3\rangle\langle 1|) + (|3\rangle\langle 1|))$

In[25]:=

```
H // MatrixForm
```

Out[25]//MatrixForm=

$$\begin{pmatrix} 0 & e^{i\alpha} k & 0 \\ e^{i\alpha} k & 0 & e^{i\alpha} k \\ 2 e^{i\alpha} k & e^{i\alpha} k & 0 \end{pmatrix}$$

In[26]:=

```
< 3 | H | 1 >
```

Out[26]= $2 e^{i\alpha} k$

In[27]:=

```
< 2 | H | 3 >
```

Out[27]= $e^{i\alpha} k$

In[28]:=

```
< 1 | H | 2 >
```

Out[28]= $e^{i\alpha} k$

In[29]:=

```
e^(-i H t) // MatrixForm
```

Out[29]//MatrixForm=

$$\begin{pmatrix} 1 & e^{-i e^{i\alpha} k t} & 1 \\ e^{-i e^{i\alpha} k t} & 1 & e^{-i e^{i\alpha} k t} \\ e^{-2 i e^{i\alpha} k t} & e^{-i e^{i\alpha} k t} & 1 \end{pmatrix}$$

In[30]:=

```
e^(-i H 2 t) // MatrixForm
```

Out[30]//MatrixForm=

$$\begin{pmatrix} 1 & e^{-2 i e^{i\alpha} k t} & 1 \\ e^{-2 i e^{i\alpha} k t} & 1 & e^{-2 i e^{i\alpha} k t} \\ e^{-4 i e^{i\alpha} k t} & e^{-2 i e^{i\alpha} k t} & 1 \end{pmatrix}$$

In[31]:=

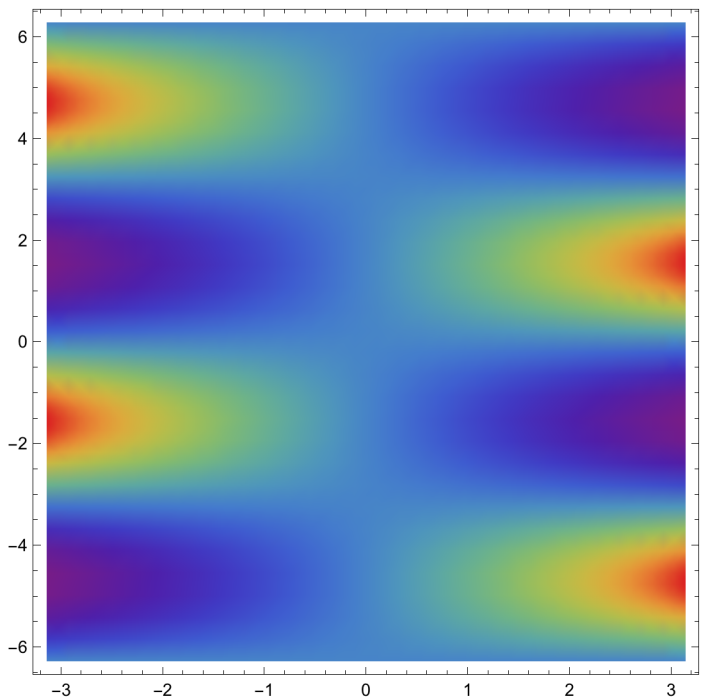
```
< 3 | e^(-i H t) | 1 > /. {k * t → (θ / (4 * Pi))}
```

Out[31]= $e^{-\frac{i e^{i\alpha} \theta}{2\pi}}$

$$\left\| \langle 3 \mid e^{-i H t} \mid 1 \rangle \right\|^2$$

In[32]:=
```
DensityPlot[Norm[ < 3 | e^(-i H t) | 1 > /. {k * t → (θ / (4 * Pi))}]^2,
  {θ,  -Pi, Pi}, {α, -2 * Pi, 2 * Pi}, ColorFunction → "Rainbow", PlotPoints → 30]
```

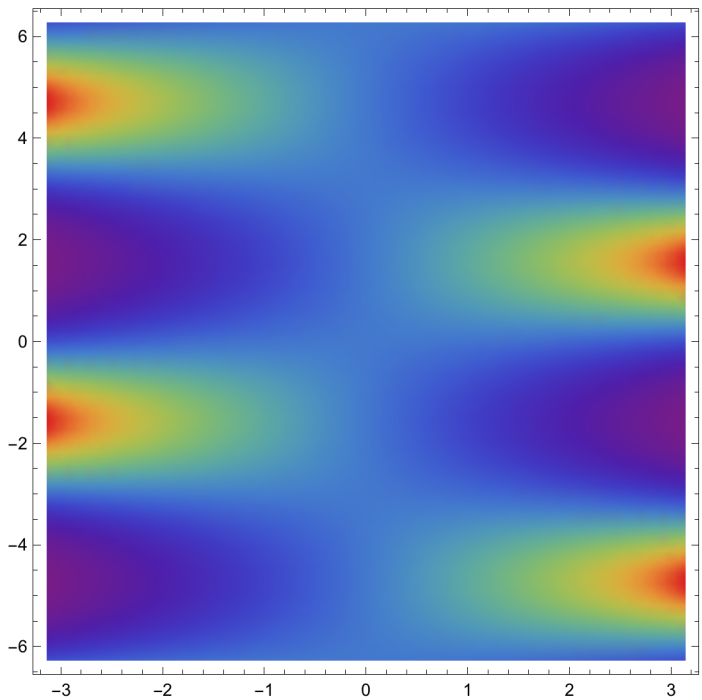
Out[32]=



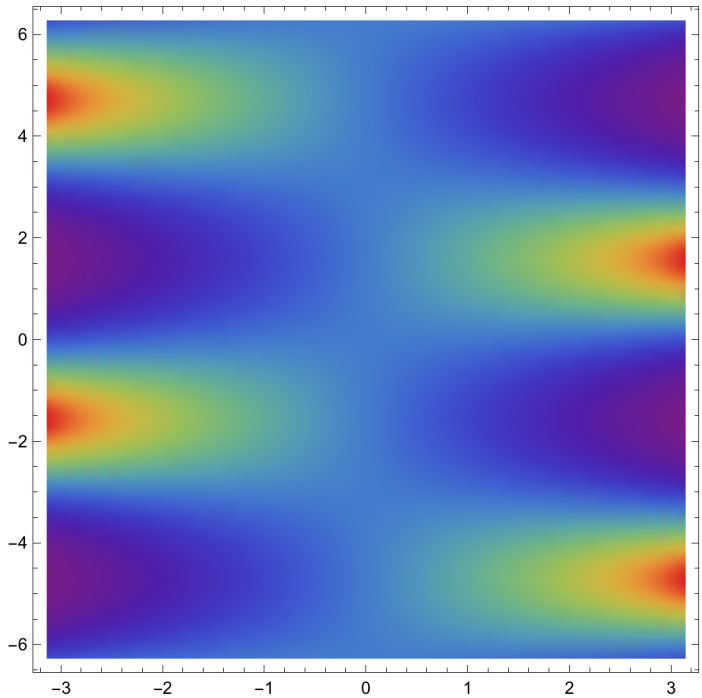
$$\left\| \langle 3 \mid U^{1,2}\, U^{2,3}\, U^{3,1}\, U^{3,1}\, U^{1,2} \mid 1 \rangle \right\|^2$$

In[33]:=
```
DensityPlot[Norm[U^{1,2} U^{2,3} U^{3,1} U^{3,1} U^{1,2} /. {k * t → (θ / (4 * Pi))}]^2,
  {θ, -Pi, Pi}, {α, -2 * Pi, 2 * Pi}, ColorFunction → "Rainbow", PlotPoints → 30]
```

Out[33]=



$$\left\| \langle 3 \mid U^{1,2}\, U^{2,3}\, U^{3,1}\, U^{3,1}\, U^{2,3}\, U^{1,2} \mid 1 \rangle \right\|^2$$

In[34]:=
```
DensityPlot[Norm[U^{1,2} U^{2,3} U^{3,1} U^{3,1} U^{2,3} U^{1,2} /. {k * t → (θ / (4 * Pi))}]^2,
  {θ, -Pi, Pi}, {α, -2 * Pi, 2 * Pi}, ColorFunction → "Rainbow", PlotPoints → 30]
```

Out[34]=