Catwalk Mechana

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References

Blog: http://www.azimuthproject.org/azimuth/show/Blog%20-%20time%20inversion%20symmetry%20breaking%20%28version%202%29

Zoltán Zimborás, Mauro Faccin, Zoltán Kádár, James Whitfield, Ben Lanyon and Jacob Biamonte, Quantum Transport Enhancement by Time-Reversal Symmetry Breaking, Scientific Reports 3, 2361 (2013).

```
<< Notation`
```

```
outerFun := TensorProduct
Notation[ x_m_,n_  ⟺  x_[[m_]][[n_]] ]
Notation[ | x_ > < y_ |  ⟹
  If[StringMatchQ[ToString[FullForm[sites[[y_]]]], "*CirclePlus*"] == True,
   Outer[outerFun, sites[[x_]], Distribute[Conjugate[sites[[y_]]], CirclePlus]],

   Outer[outerFun, sites[[x_]], Conjugate[sites[[y_]]]]] ]
Notation[ σˣ  ⟹  PauliMatrix[1] ]
Notation[ σʸ  ⟹  PauliMatrix[2] ]
Notation[ σᶻ  ⟹  PauliMatrix[3] ]
Notation[ ∑^sites expr_  ⟹  Sum[expr_, {m, 1, Length[sites]}, {n, 1, Length[sites]}] ]
```

Let’s verify the symbolics are correctly done by setting up some simple setup.

Outer Product of Ket and Bra

Most important is $|m\rangle\langle n|$, where sites is an array holding values related to Spin, the integers m and n index the Hilbert space vectors, the Outer product is linear map:

$|\phi\rangle\langle\psi| \;:\; |o\rangle \longrightarrow |\phi\rangle\langle\psi|o\rangle$ **(EQ 1)**

$$|\phi\rangle\langle\psi| \doteq \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{pmatrix} \begin{pmatrix} \psi_1^* & \psi_2^* & \cdots & \psi_N^* \end{pmatrix} = \begin{pmatrix} \phi_1\psi_1^* & \phi_1\psi_2^* & \cdots & \phi_1\psi_N^* \\ \phi_2\psi_1^* & \phi_2\psi_2^* & \cdots & \phi_2\psi_N^* \\ \vdots & \vdots & \ddots & \vdots \\ \phi_N\psi_1^* & \phi_N\psi_2^* & \cdots & \phi_N\psi_N^* \end{pmatrix}$$

Somehow indicating a measure of projection for subspace spanned by $|\phi\rangle$.

Simple binary vectors to test the code. //MatrixForm prints Tensors in matrix format:

```
sites = {{1, 0}, {0, 1}};
| 1 > < 2 | // MatrixForm
```

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

```
| 2 > < 1 | // MatrixForm
```

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

```
| 1 > < 1 | // MatrixForm
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

```
| 2 > < 2 | // MatrixForm
```

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

For Pauli matrices:

```
sites = {σˣ, σʸ};
σˣ // MatrixForm

σʸ // MatrixForm
```

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Notice same Bra-Ket notation, though the algebra changed, works! the operators are properly abstracted in the *Mathematica* code, note that the MatrixForm is printing Tensors, in books and websites they print these as 4x4 matrices:

$|\sigma^x\rangle\langle\sigma^y|$

```
|1 > <2| // MatrixForm
```

$$\begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}$$

$|\sigma^y\rangle\langle\sigma^x|$

```
|2 > <1| // MatrixForm
```

$$\begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}$$

$|\sigma^x\rangle\langle\sigma^x|$

```
|1 > <1| // MatrixForm
```

$$\begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}$$

$|\sigma^y\rangle\langle\sigma^y|$

```
|2 > <2| // MatrixForm
```

$$\begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}$$

⊕

I suspect the physics papers use + the Tensor summation instead of ⊕ which I understand the algebra. So I took Zoltan's computations and tried to duplicate this equations (1-3) and if I use the ⊕ then I understand what happens, if I use + no idea what is happening!

Remark 1: *⊕ has no meaning, called **CirclePlus**, it is an abstract sum operator. However we could program it to be any specific summation.*

```
sites = {σˣ ⊕ σʸ, σˣ ⊕ σʸ};
 | 1 >  < 1 | // MatrixForm
```

```
({{{{0, 0}, {0, 0}}, {{0, 1}, {1, 0}}}, {{{0, 1}, {1, 0}}, {{0, 0}, {0, 0}}}} ⊕
   {{{{0, 0}, {0, 0}}, {{0, i}, {-i, 0}}}, {{{0, i}, {-i, 0}}, {{0, 0}, {0, 0}}}}) ⊕
 ({{{{0, 0}, {0, 0}}, {{0, -i}, {-i, 0}}}, {{{0, i}, {i, 0}}, {{0, 0}, {0, 0}}}} ⊕
   {{{{0, 0}, {0, 0}}, {{0, 1}, {-1, 0}}}, {{{0, -1}, {1, 0}}, {{0, 0}, {0, 0}}}})
```

Which the latter is in matrix form:

(EQ 2)

$$\begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \oplus \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \oplus \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \oplus \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}$$

Or rewrite in terms of the original basis:

(EQ 3)

$$|\,\sigma^x\rangle\langle\sigma^x\,| + |\,\sigma^x\rangle\langle\sigma^y\,| + |\,\sigma^y\rangle\langle\sigma^x\,| + |\,\sigma^y\rangle\langle\sigma^y\,|$$

H_{CQW}

The expression verbatim from the paper, actually runs in *Mathematica*, with 'sites' could contain any array of information about Spin:

(EQ 4)

$$HCQW = \sum^{sites} \left(J_{n,m}\, e^{i\,\theta_{n,m}} \left(|\,m> \;<n\,|\right) + J_{n,m}\, e^{i\,\theta_{n,m}} \left(|\,n> \;<m\,|\right)\right)$$

$$\begin{pmatrix} \begin{pmatrix} -5.87167 - 1.23432\,i & 3.45776 - 3.25534\,i \\ 6.24979 + 1.95977\,i & 5.87167 + 1.23432\,i \end{pmatrix} & \begin{pmatrix} 6.24979 + 1.95977\,i & -12.1137 - 7.99699\,i \\ 11.6187 + 20.9295\,i & -6.24979 - 1.95977\,i \end{pmatrix} \\ \begin{pmatrix} 3.45776 - 3.25534\,i & -2.7125 - 6.03276\,i \\ -12.1137 - 7.99699\,i & -3.45776 + 3.25534\,i \end{pmatrix} & \begin{pmatrix} 5.87167 + 1.23432\,i & -3.45776 + 3.25534\,i \\ -6.24979 - 1.95977\,i & -5.87167 - 1.23432\,i \end{pmatrix} \end{pmatrix}$$

Subscripts produce numbers which for now are picked from a random list, but could program any function of course :

$\theta_{1,2}$

0.981214

$J_{3,2}$

3

CODE

```
sites = Table[PauliMatrix[RandomInteger[{1, 3}]], {i, 1, 5}];
θ = Table[RandomReal[{0, 2 * Pi}], {i, 1, 5}, {j, 1, 5}];
J = Table[RandomInteger[{1, 3}], {i, 1, 5}, {j, 1, 5}];
```