Catwalk: Topology & Laplacian

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References

Blog: http://www.azimuthproject.org/azimuth/show/Blog%20-%20time%20inversion%20symmetry%20breaking%20%28version%202%29

Zoltán Zimborás, Mauro Faccin, Zoltán Kádár, James Whitfield, Ben Lanyon and Jacob Biamonte, Quantum Transport Enhancement by Time-Reversal Symmetry Breaking, Scientific Reports 3, 2361 (2013).

```
makeTOP[list_] := Graph[list, VertexLabels → "Name", EdgeStyle → Arrowheads[.08]]
makeDEGREE[g_] := Module[{vlist, n},
  vlist = VertexList[g];
  n = Length[vlist];
  Table[If[i == j, VertexDegree[g, vlist[[i]]], 0], {i, 1, n}, {j, 1, n}]
 ]
(* weighted adjacency *)
makeDEGREEw[A_] :=
 DiagonalMatrix[Table[Total[Table[A[[i]][[j]], {j, 1, Dimensions[A][[1]]}]],
   {i, 1, Dimensions[A][[1]]}]]
makeψ[n_] := Table[ψi[t], {i, 1, n}]
```

```
ladder = Table[i ⟷ (i + 1), {i, 1, 6 - 1}]
```

```
{1 ⟷ 2, 2 ⟷ 3, 3 ⟷ 4, 4 ⟷ 5, 5 ⟷ 6}
```

Topology: Ladder

Make a linear bi-directional graph, no arrowheads means bi-directional:

```
top = makeTOP[ladder]
𝒜 = AdjacencyMatrix[top];
𝒜 // MatrixForm
```

0	1	0	0	0	0
1	0	1	0	0	0
0	1	0	1	0	0
0	0	1	0	1	0
0	0	0	1	0	1
0	0	0	0	1	0

```
𝒟 = makeDEGREE[top];
𝒟 // MatrixForm
```

1	0	0	0	0	0
0	2	0	0	0	0
0	0	2	0	0	0
0	0	0	2	0	0
0	0	0	0	2	0
0	0	0	0	0	1

Classical Catwalk: Laplacian

```
ℒ = 𝒟 - 𝒜;
ℒ // MatrixForm
```

1	-1	0	0	0	0
-1	2	-1	0	0	0
0	-1	2	-1	0	0
0	0	-1	2	-1	0
0	0	0	-1	2	-1
0	0	0	0	-1	1

These are setups for future solvers:

```
Ψ = makeψ[6]
```

$\{\psi_1[t], \psi_2[t], \psi_3[t], \psi_4[t], \psi_5[t], \psi_6[t]\}$

```
-ℒ.Ψ
```

$\{-\psi_1[t] + \psi_2[t], \psi_1[t] - 2\,\psi_2[t] + \psi_3[t], \psi_2[t] - 2\,\psi_3[t] + \psi_4[t],$
$\psi_3[t] - 2\,\psi_4[t] + \psi_5[t], \psi_4[t] - 2\,\psi_5[t] + \psi_6[t], \psi_5[t] - \psi_6[t]\}$

```
Dt[Ψ, t]
```

$\{\psi_1'[t], \psi_2'[t], \psi_3'[t], \psi_4'[t], \psi_5'[t], \psi_6'[t]\}$

Exponentiate to Shift Time

$\partial_t \psi(t) = -L\, \psi(t) \Longrightarrow e^{\partial_t}\, \psi(t) = e^{-L}\, \psi(t)$ (EQ 1)

e^{-L}

```
expL = Simplify[MatrixExp[-ℒ]];
Simplify[expL] // MatrixForm
```

$$\left(\begin{array}{ll}
\frac{1}{12}\left(2+\frac{1}{e^3}+\frac{2}{e^2}+\frac{3}{e}+\frac{e^{-2-\sqrt{3}}}{2+\sqrt{3}}-\frac{e^{-2+\sqrt{3}}}{-2+\sqrt{3}}\right) & \frac{1}{12}e^{-3-\sqrt{3}}\left(e-\sqrt{3}\,e-2 \cdots\right. \\
\frac{1}{12}e^{-3-\sqrt{3}}\left(e-\sqrt{3}\,e-2\,e^{\sqrt{3}}-2\,e^{1+\sqrt{3}}+2\,e^{3+\sqrt{3}}+\left(1+\sqrt{3}\right)e^{1+2\sqrt{3}}\right) & \frac{1}{6}\left(1 \cdots\right. \\
\frac{1}{12}\left(2+\frac{1}{e^3}-\frac{2}{e^2}-\frac{3}{e}+e^{-2-\sqrt{3}}+e^{-2+\sqrt{3}}\right) & \frac{1}{12}e^{-3-\sqrt{3}}\left(-\left(1+\sqrt{3}\right)e- \cdots\right. \\
\frac{1}{12}\left(2+\frac{1}{e^3}+\frac{2}{e^2}-\frac{3}{e}-e^{-2-\sqrt{3}}-e^{-2+\sqrt{3}}\right) & -\frac{1}{12}e^{-3-\sqrt{3}}\left(-\left(1+\sqrt{3}\right)e+ \cdots\right. \\
-\frac{1}{12}e^{-3-\sqrt{3}}\left(e-\sqrt{3}\,e+2\,e^{\sqrt{3}}-2\,e^{1+\sqrt{3}}-2\,e^{3+\sqrt{3}}+\left(1+\sqrt{3}\right)e^{1+2\sqrt{3}}\right) & \frac{1}{6}\left(1 \cdots\right. \\
\frac{1}{12}\left(2+\frac{1}{e^3}-\frac{2}{e^2}+\frac{3}{e}-\frac{e^{-2-\sqrt{3}}}{2+\sqrt{3}}+\frac{e^{-2+\sqrt{3}}}{-2+\sqrt{3}}\right) & -\frac{1}{12}e^{-3-\sqrt{3}}\left(e-\sqrt{3}\,e+ \cdots\right.
\end{array}\right.$$

```
N[expL] // MatrixForm
```

0.523778	0.308508	0.122031	0.0356607	0.00822928	0.00179328
0.308508	0.3373	0.222138	0.0945992	0.0292248	0.00822928
0.122031	0.222138	0.309868	0.215702	0.0945992	0.0356607
0.0356607	0.0945992	0.215702	0.309868	0.222138	0.122031
0.00822928	0.0292248	0.0945992	0.222138	0.3373	0.308508
0.00179328	0.00822928	0.0356607	0.122031	0.308508	0.523778

e^{∂_t}

Let’s look at Taylor series of arbitrary function centered at ‘a’ for shift ‘y’ along t-axis:

```
expr = Series[f[t + y], {y, a, 10}]
```

$$f[a+t] + f'[a+t]\,(y-a) + \frac{1}{2} f''[a+t]\,(y-a)^2 + \frac{1}{6} f^{(3)}[a+t]\,(y-a)^3 + \frac{1}{24} f^{(4)}[a+t]\,(y-a)^4 + \frac{1}{120} f^{(5)}[a+t]\,(y-a)^5 + \frac{1}{720} f^{(6)}[a+t]\,(y-a)^6 + \frac{f^{(7)}[a+t]\,(y-a)^7}{5040} + \frac{f^{(8)}[a+t]\,(y-a)^8}{40\,320} + \frac{f^{(9)}[a+t]\,(y-a)^9}{362\,880} + \frac{f^{(10)}[a+t]\,(y-a)^{10}}{3\,628\,800} + O[y-a]^{11}$$

(EQ 2)

```
Normal[Normal[expr] /. {y → a + δ}] /. {a → 0}
```

$$f[t] + \delta\, f'[t] + \frac{1}{2}\delta^2 f''[t] + \frac{1}{6}\delta^3 f^{(3)}[t] + \frac{1}{24}\delta^4 f^{(4)}[t] + \frac{1}{120}\delta^5 f^{(5)}[t] + \frac{1}{720}\delta^6 f^{(6)}[t] + \frac{\delta^7 f^{(7)}[t]}{5040} + \frac{\delta^8 f^{(8)}[t]}{40\,320} + \frac{\delta^9 f^{(9)}[t]}{362\,880} + \frac{\delta^{10} f^{(10)}[t]}{3\,628\,800}$$

(EQ 3)

This a exponentiation and of course the original t-shift:

$e^{\delta\,\partial_t}(f(t)) = f(t+\delta)$ (EQ 4)

In other words $e^{\delta\,\partial_t}$ is a shift operator.

Assumptions: *f and all its derivatives are well-behaved between 0 and δ.*

In particular for $\delta = 1$:

$e^{\partial_t} f(t) = f(t+1)$ (EQ 5)

Therefore use the time-shift:

$e^{\partial_t}\,\psi(t) = e^{-L}\,\psi(t) \Longrightarrow \psi(t+1) = e^{-L}\,\psi(t)$ or $\psi(t+\delta) = e^{-\delta L}\,\psi(t)$ (EQ 6)

Infinite Steps

Reach e^{-L} to a very large power:

```
MatrixPower[N[expL], 2 000 000] // MatrixForm
```

$$\begin{pmatrix} 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 \\ 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 \\ 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 \\ 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 \\ 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 \\ 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 \end{pmatrix}$$

Or

$$\left(e^{-L}\right)^{\infty} = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

Therefore the probability of Cat on any rung after a long passage of time is a uniform distribution i.e. fully symmetric.

Longer Ladder

```
ladder = Table[i ⟷ (i + 1), {i, 1, 10 - 1}]
top = makeTOP[ladder]
```

```
{1 ⟷ 2, 2 ⟷ 3, 3 ⟷ 4, 4 ⟷ 5, 5 ⟷ 6, 6 ⟷ 7, 7 ⟷ 8, 8 ⟷ 9, 9 ⟷ 10}
```

```
𝒜 = AdjacencyMatrix[top];
𝒟 = makeDEGREE[top];
ℒ = 𝒟 - 𝒜;
expL = MatrixExp[-ℒ];
MatrixPower[N[expL], 2 000 000] // MatrixForm
```

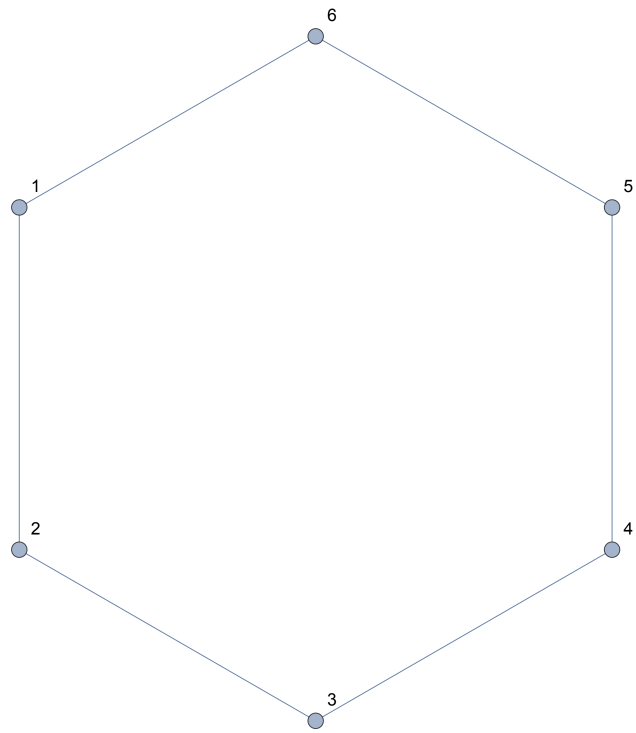
$$\begin{pmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \end{pmatrix}$$

Same uniform distribution but this time amongst 10 steps.

Loop Topology

Bi-Directional loop:

```
circle = {1 ⟷ 2, 2 ⟷ 3, 3 ⟷ 4, 4 ⟷ 5, 5 ⟷ 6, 6 ⟷ 1};
top = makeTOP[circle]
```




```
𝒜 = AdjacencyMatrix[top];
𝒟 = makeDEGREE[top];
ℒ = 𝒟 - 𝒜;
ℒ // MatrixForm
```

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

```
expL = N[MatrixExp[-ℒ]];
MatrixPower[N[expL], 500] // MatrixForm
```

$$\begin{pmatrix} 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 \\ 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 \\ 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 \\ 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 \\ 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 \\ 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 \end{pmatrix}$$

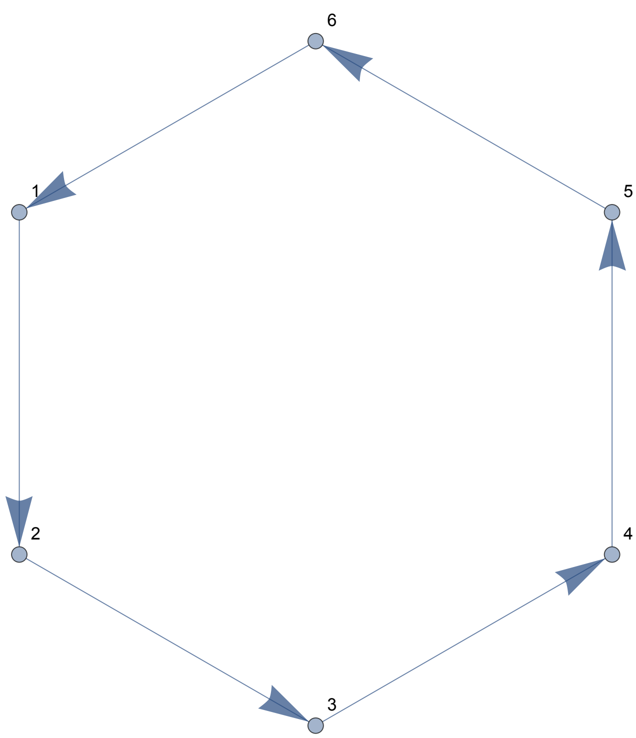
This is actually

$$\begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

Again same uniform distribution.

1-Directional Loop

```
circle = {1 -> 2, 2 -> 3, 3 -> 4, 4 -> 5, 5 -> 6, 6 -> 1};
top = makeTOP[circle]
```



```
𝒜 = AdjacencyMatrix[top];
𝒟 = makeDEGREE[top];
ℒ = 𝒟 - 𝒜;
ℒ // MatrixForm
```

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ -1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

```
expL = N[MatrixExp[-ℒ]];
MatrixPower[N[expL], 800] // MatrixForm
```

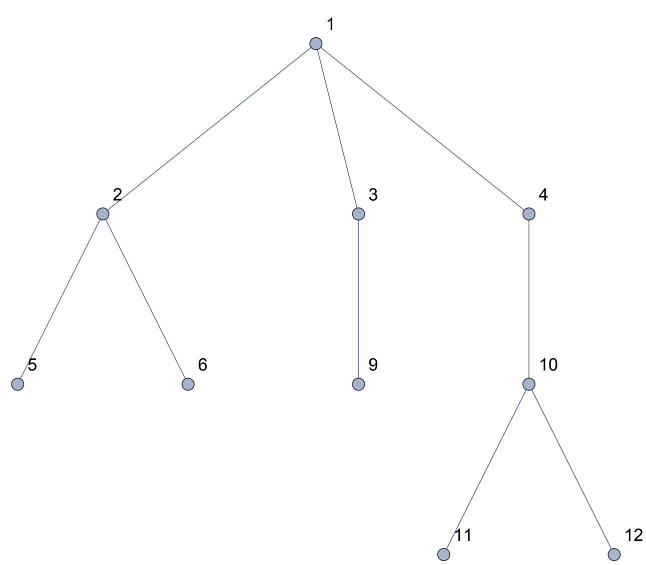
$$\begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

We get uniform distribution of 0, effectively this is the infinite line and the cat walks to oblivion.

Others

The following examples show uniform distribution computed no matter what the topology:

```
dandi = {1 ⟷ 2, 1 ⟷ 3, 1 ⟷ 4, 2 ⟷ 5, 2 ⟷ 6, 3 ⟷ 9, 4 ⟷ 10, 10 ⟷ 11, 10 ⟷ 12};
top = makeTOP[dandi]
```



```
𝒜 = AdjacencyMatrix[top];
𝒟 = makeDEGREE[top];
ℒ = 𝒟 - 𝒜;
ℒ // MatrixForm
```

$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 3 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 3 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

```
expL = N[MatrixExp[-ℒ]];
```

```
Style[expL // MatrixForm, 7]
```

0.192644	0.126263	0.162564	0.153221	0.0793477	0.0793477	0.0972553	0.0597497	0.0248037	0.0248037
0.126263	0.23015	0.0614401	0.0597497	0.235016	0.235016	0.0251385	0.0168155	0.00520563	0.00520563
0.162564	0.0614401	0.317193	0.0738306	0.0251385	0.0251385	0.302461	0.0201318	0.00605085	0.00605085
0.153221	0.0597497	0.0738306	0.272034	0.0248037	0.0248037	0.0294756	0.165881	0.0981005	0.0981005
0.0793477	0.235016	0.0251385	0.0248037	0.494357	0.126477	0.00723089	0.00520563	0.00121155	0.00121155
0.0793477	0.235016	0.0251385	0.0248037	0.126477	0.494357	0.00723089	0.00520563	0.00121155	0.00121155
0.0972553	0.0251385	0.302461	0.0294756	0.00723089	0.00723089	0.522399	0.00605085	0.00137895	0.00137895
0.0597497	0.0168155	0.0201318	0.165881	0.00520563	0.00520563	0.00605085	0.242605	0.239178	0.239178
0.0248037	0.00520563	0.00605085	0.0981005	0.00121155	0.00121155	0.00137895	0.239178	0.49537	0.12749
0.0248037	0.00520563	0.00605085	0.0981005	0.00121155	0.00121155	0.00137895	0.239178	0.12749	0.49537

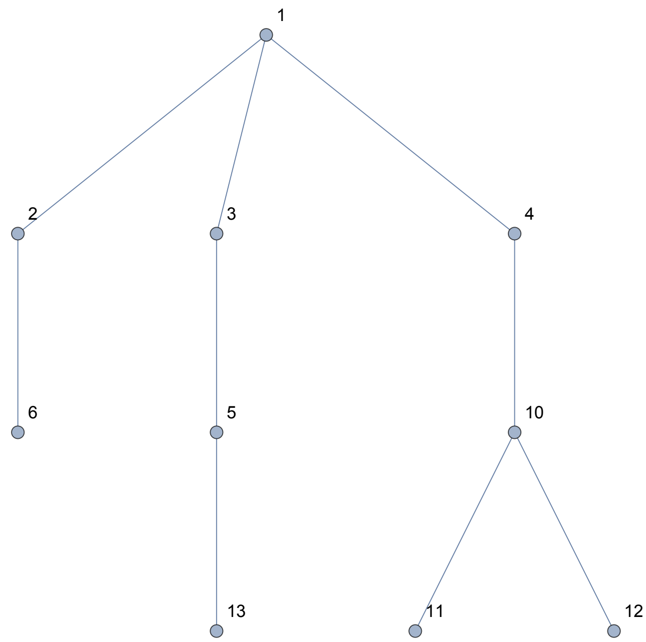
```
MatrixPower[N[expL], 500] // MatrixForm
```

$$\begin{pmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \end{pmatrix}$$

Again uniform distribution.

Another...

```
dandi = {1 ⟷ 2, 1 ⟷ 3,  1 ⟷ 4, 2 ⟷ 6, 3 ⟷ 5, 4 ⟷ 10, 10 ⟷ 11, 10 ⟷ 12, 5 ⟷ 13};
top = makeTOP[dandi]
```




```
𝒜 = AdjacencyMatrix[top];
𝒟 = makeDEGREE[top];
ℒ = 𝒟 - 𝒜;
ℒ // MatrixForm
```

3	−1	−1	−1	0	0	0	0	0	0
−1	2	0	0	−1	0	0	0	0	0
−1	0	2	0	0	−1	0	0	0	0
−1	0	0	2	0	0	−1	0	0	0
0	−1	0	0	1	0	0	0	0	0
0	0	−1	0	0	2	0	0	0	−1
0	0	0	−1	0	0	3	−1	−1	0
0	0	0	0	0	0	−1	1	0	0
0	0	0	0	0	0	−1	0	1	0
0	0	0	0	0	−1	0	0	0	1

```
(* this took a long time to exponentiate!!!!*)
expL = N[MatrixExp[-ℒ]];

Style[expL // MatrixForm, 7]
```

0.203902	0.165827	0.160337	0.156451	0.0980928	0.0753709	0.0603683	0.0249323	0.0249323	0.0297857
0.165827	0.317909	0.0753709	0.0745422	0.302607	0.0242955	0.0202439	0.00607009	0.00607009	0.00706377
0.160337	0.0753709	0.290802	0.0734374	0.0297857	0.217233	0.020079	0.00604316	0.00604316	0.120869
0.156451	0.0745422	0.0734374	0.272742	0.0296207	0.0239656	0.165992	0.0981197	0.0981197	0.00700992
0.0980928	0.302607	0.0297857	0.0296207	0.522423	0.00706377	0.00607009	0.00138172	0.00138172	0.00157362
0.0753709	0.0242955	0.217233	0.0239656	0.00706377	0.3363	0.00507641	0.00118982	0.00118982	0.308316
0.0603683	0.0202439	0.020079	0.165992	0.00607009	0.00507641	0.24262	0.23918	0.23918	0.00118982
0.0249323	0.00607009	0.00604316	0.0981197	0.00138172	0.00118982	0.23918	0.49537	0.12749	0.000223067
0.0249323	0.00607009	0.00604316	0.0981197	0.00138172	0.00118982	0.23918	0.12749	0.49537	0.000223067
0.0297857	0.00706377	0.120869	0.00700992	0.00157362	0.308316	0.00118982	0.000223067	0.000223067	0.523747

```
MatrixPower[N[expL], 500] // MatrixForm
```

$$\begin{pmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \end{pmatrix}$$

Uniform distribution.

Weighted Adjacency Matrix

```
p =.;
top = Graph[{1 → 2, 2 → 3, 3 → 4, 4 → 5, 5 → 6, 2 → 1, 3 → 2, 4 → 3, 5 → 4, 6 → 5},
  EdgeWeight → {1 + p, 1 + p, 1 + p, 1 + p, 1 + p, 1 - p, 1 - p, 1 - p, 1 - p, 1 - p}]
```

Test the weights:

```
𝒜 = WeightedAdjacencyMatrix[top];
𝒜 // MatrixForm
```

$$\begin{pmatrix} 0 & 1+p & 0 & 0 & 0 & 0 \\ 1-p & 0 & 1+p & 0 & 0 & 0 \\ 0 & 1-p & 0 & 1+p & 0 & 0 \\ 0 & 0 & 1-p & 0 & 1+p & 0 \\ 0 & 0 & 0 & 1-p & 0 & 1+p \\ 0 & 0 & 0 & 0 & 1-p & 0 \end{pmatrix}$$

P = 1

```
p = 1;
top = Graph[{1 → 2, 2 → 3, 3 → 4, 4 → 5, 5 → 6, 2 → 1, 3 → 2, 4 → 3, 5 → 4, 6 → 5},
   EdgeWeight → {1 + p, 1 + p, 1 + p, 1 + p, 1 + p, 1 - p, 1 - p, 1 - p, 1 - p, 1 - p}];
𝒜 = WeightedAdjacencyMatrix[top];
𝒟 = makeDEGREEw[𝒜];
ℒ = 𝒟 - 𝒜;
ℒ // MatrixForm
```

$$\begin{pmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 2 & -2 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
expL = N[MatrixExp[-ℒ]];
Style[expL // MatrixForm, 7]
```

$$\begin{pmatrix} 0.135335 & 0.270671 & 0.270671 & 0.180447 & 0.0902235 & 0.052653 \\ 0. & 0.135335 & 0.270671 & 0.270671 & 0.180447 & 0.142877 \\ 0. & 0. & 0.135335 & 0.270671 & 0.270671 & 0.323324 \\ 0. & 0. & 0. & 0.135335 & 0.270671 & 0.593994 \\ 0. & 0. & 0. & 0. & 0.135335 & 0.864665 \\ 0. & 0. & 0. & 0. & 0. & 1. \end{pmatrix}$$

```
MatrixPower[N[expL], 500] // MatrixForm
```

$$\begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 1. \\ 0. & 0. & 0. & 0. & 0. & 1. \\ 0. & 0. & 0. & 0. & 0. & 1. \\ 0. & 0. & 0. & 0. & 0. & 1. \\ 0. & 0. & 0. & 0. & 0. & 1. \\ 0. & 0. & 0. & 0. & 0. & 1. \end{pmatrix}$$

The cat ends up at the right-most rung 6.

$p = \frac{1}{2}$

```
p = 1 / 2;
top = Graph[{1 → 2, 2 → 3, 3 → 4, 4 → 5, 5 → 6, 2 → 1, 3 → 2, 4 → 3, 5 → 4, 6 → 5},
   EdgeWeight → {1 + p, 1 + p, 1 + p, 1 + p, 1 + p, 1 - p, 1 - p, 1 - p, 1 - p, 1 - p}];
𝒜 = WeightedAdjacencyMatrix[top];
𝒟 = makeDEGREEw[𝒜];
ℒ = 𝒟 - 𝒜;
ℒ // MatrixForm
```

$$\begin{pmatrix} \frac{3}{2} & -\frac{3}{2} & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 2 & -\frac{3}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 2 & -\frac{3}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 2 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 2 & -\frac{3}{2} \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

```
expL = N[MatrixExp[-ℒ]];
Style[expL // MatrixForm, 7]
```

$$\begin{pmatrix} 0.303142 & 0.330902 & 0.216435 & 0.10022 & 0.0359722 & 0.013328 \\ 0.110301 & 0.264987 & 0.292164 & 0.195019 & 0.0926721 & 0.0448575 \\ 0.0240484 & 0.0973878 & 0.257848 & 0.289648 & 0.197981 & 0.133087 \\ 0.00371186 & 0.0216688 & 0.0965492 & 0.258835 & 0.303119 & 0.316116 \\ 0.000444101 & 0.0034323 & 0.0219979 & 0.10104 & 0.298213 & 0.574872 \\ 0.0000548478 & 0.000553797 & 0.00492915 & 0.035124 & 0.191624 & 0.767714 \end{pmatrix}$$

```
MatrixPower[N[expL], 500] // MatrixForm
```

$$\begin{pmatrix} 0.00274725 & 0.00824176 & 0.0247253 & 0.0741758 & 0.222527 & 0.667582 \\ 0.00274725 & 0.00824176 & 0.0247253 & 0.0741758 & 0.222527 & 0.667582 \\ 0.00274725 & 0.00824176 & 0.0247253 & 0.0741758 & 0.222527 & 0.667582 \\ 0.00274725 & 0.00824176 & 0.0247253 & 0.0741758 & 0.222527 & 0.667582 \\ 0.00274725 & 0.00824176 & 0.0247253 & 0.0741758 & 0.222527 & 0.667582 \\ 0.00274725 & 0.00824176 & 0.0247253 & 0.0741758 & 0.222527 & 0.667582 \end{pmatrix}$$

Note that upper-half-triangle of matrix is larger than the lower-half-triangle of the matrix i.e. probabilities to the right are larger.

Quantum Catwalk

```
ladder = Table[i ⟷ (i + 1), {i, 1, 6 - 1}]
top = makeTOP[ladder]
```

{1 ⟷ 2, 2 ⟷ 3, 3 ⟷ 4, 4 ⟷ 5, 5 ⟷ 6}

```
𝒜 = AdjacencyMatrix[top];
𝒟 = makeDEGREE[top];
ℒ = 𝒟 - 𝒜;
ℒ // MatrixForm
```

1	-1	0	0	0	0
-1	2	-1	0	0	0
0	-1	2	-1	0	0
0	0	-1	2	-1	0
0	0	0	-1	2	-1
0	0	0	0	-1	1

```
expLi = Simplify[N[MatrixExp[ⅈ * ℒ]]];
Style[Simplify[expLi] // MatrixForm, 7]
```

0.431243 + 0.443586 i	0.671245 - 0.0808291 i	0.0295854 - 0.37449 i	-0.131402 - 0.0227259 i	-0.0079144 + 0.0337891 i	0.00724282 + 0.000669986 i
0.671245 - 0.0808291 i	-0.210417 + 0.149925 i	0.510258 + 0.270935 i	0.153073 - 0.317975 i	-0.116245 - 0.055845 i	-0.0079144 + 0.0337891 i
0.0295854 - 0.37449 i	0.510258 + 0.270935 i	-0.0869294 + 0.20644 i	0.525415 + 0.237815 i	0.153073 - 0.317975 i	-0.131402 - 0.0227259 i
-0.131402 - 0.0227259 i	0.153073 - 0.317975 i	0.525415 + 0.237815 i	-0.0869294 + 0.20644 i	0.510258 + 0.270935 i	0.0295854 - 0.37449 i
-0.0079144 + 0.0337891 i	-0.116245 - 0.055845 i	0.153073 - 0.317975 i	0.510258 + 0.270935 i	-0.210417 + 0.149925 i	0.671245 - 0.0808291 i
0.00724282 + 0.000669986 i	-0.0079144 + 0.0337891 i	-0.131402 - 0.0227259 i	0.0295854 - 0.37449 i	0.671245 - 0.0808291 i	0.431243 + 0.443586 i

Infinite Steps: No Convergence

• No convergence. In a stochastic walk, if you leave the walker to wander for a long time, eventually the probability of finding a walker at a node converges to a constant value. In a quantum walk, this doesn't happen, so the walk can't be characterized so easily by its long-time properties.

Source: http://johncarlosbaez.wordpress.com/2013/08/05/quantum-network-theory-part-1/

Try 20,000,000 steps or 40,000,000 no convergence, not even the norms of the elements of the power:

```
expLiINF = MatrixPower[N[expLi], 20 000 000];
Style[expLiINF // MatrixForm, 7]
Style[Table[Norm[expLiINF[[i]][[j]]], {i, 1, Dimensions[expLiINF][[1]]},
   {j, 1, Dimensions[expLiINF][[2]]}] // MatrixForm, 7]
```

0.257358 - 0.726578 i	0.391758 + 0.0785732 i	-0.0653668 + 0.154879 i	-0.0676218 + 0.141369 i	0.22794 + 0.0920325 i	0.255932 + 0.259725 i
0.391758 + 0.0785732 i	-0.199766 - 0.650273 i	0.389503 + 0.0650638 i	0.230195 + 0.105542 i	-0.0396298 + 0.309061 i	0.22794 + 0.0920325 i
-0.0653668 + 0.154879 i	0.389503 + 0.0650638 i	0.0957956 - 0.699609 i	0.417495 + 0.232756 i	0.230195 + 0.105542 i	-0.0676218 + 0.141369 i
-0.0676218 + 0.141369 i	0.230195 + 0.105542 i	0.417495 + 0.232756 i	0.0957956 - 0.699609 i	0.389503 + 0.0650638 i	-0.0653668 + 0.154879 i
0.22794 + 0.0920325 i	-0.0396298 + 0.309061 i	0.230195 + 0.105542 i	0.389503 + 0.0650638 i	-0.199766 - 0.650273 i	0.391758 + 0.0785732 i
0.255932 + 0.259725 i	0.22794 + 0.0920325 i	-0.0676218 + 0.141369 i	-0.0653668 + 0.154879 i	0.391758 + 0.0785732 i	0.257358 - 0.726578 i

0.770811	0.39956	0.168108	0.15671	0.245818	0.364634
0.39956	0.680266	0.3949	0.253237	0.311592	0.245818
0.168108	0.3949	0.706137	0.477993	0.253237	0.15671
0.15671	0.253237	0.477993	0.706137	0.3949	0.168108
0.245818	0.311592	0.253237	0.3949	0.680266	0.39956
0.364634	0.245818	0.15671	0.168108	0.39956	0.770811

```
expLiINF = MatrixPower[N[expLi], 40 000 000];
Style[expLiINF // MatrixForm, 7]
Style[Table[Norm[expLiINF[[i]][[j]]], {i, 1, Dimensions[expLiINF][[1]]},
   {j, 1, Dimensions[expLiINF][[2]]}] // MatrixForm, 7]
```

-0.307978 - 0.176886 i	0.0616786 - 0.303833 i	0.272877 + 0.311157 i	0.222127 + 0.328586 i	0.112959 + 0.0107 i	0.638336 - 0.169724 i
0.0616786 - 0.303833 i	-0.096779 + 0.438104 i	0.0109283 - 0.286404 i	0.163709 - 0.00672851 i	0.747503 + 0.148162 i	0.112959 + 0.0107 i
0.272877 + 0.311157 i	0.0109283 - 0.286404 i	-0.205947 + 0.120218 i	0.536305 - 0.466828 i	0.163709 - 0.00672851 i	0.222127 + 0.328586 i
0.222127 + 0.328586 i	0.163709 - 0.00672851 i	0.536305 - 0.466828 i	-0.205947 + 0.120218 i	0.0109283 - 0.286404 i	0.272877 + 0.311157 i
0.112959 + 0.0107 i	0.747503 + 0.148162 i	0.163709 - 0.00672851 i	0.0109283 - 0.286404 i	-0.096779 + 0.438104 i	0.0616786 - 0.303833 i
0.638336 - 0.169724 i	0.112959 + 0.0107 i	0.222127 + 0.328586 i	0.272877 + 0.311157 i	0.0616786 - 0.303833 i	-0.307978 - 0.176886 i

0.35516	0.31003	0.413861	0.396622	0.113465	0.660514
0.31003	0.448666	0.286613	0.163848	0.762045	0.113465
0.413861	0.286613	0.238467	0.711021	0.163848	0.396622
0.396622	0.163848	0.711021	0.238467	0.286613	0.413861
0.113465	0.762045	0.163848	0.286613	0.448666	0.31003
0.660514	0.113465	0.396622	0.413861	0.31003	0.35516