CHAPTER REVIEW EXERCISE

1. True or False?

   i) $(|\mathbf{a}| \ |\mathbf{b}|)\,|\mathbf{c}| = |\mathbf{a}|\,(|\mathbf{b}|\,|\mathbf{c}|)$

   ii) $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}| \Rightarrow$ ($\mathbf{a}$ or $\mathbf{b} = \mathbf{0}$ or $\mathbf{a}$ is perpendicular to $\mathbf{b}$)

   iii) $|\mathbf{a}+\mathbf{b}| \geq |\mathbf{a}+\mathbf{c}| \Rightarrow |\mathbf{b}| \geq |\mathbf{c}|$

   iv) $|\mathbf{a}+\mathbf{b}| + |\mathbf{c}| = |\mathbf{a}| + |\mathbf{b}+\mathbf{c}|$

   v) $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}+\mathbf{c}| \Rightarrow |\mathbf{b}| = |\mathbf{c}|$

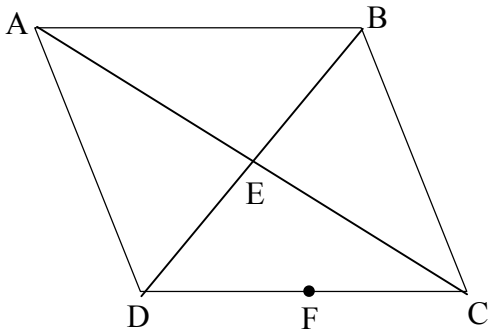
   vi) $\frac{1}{|\mathbf{a}|}\mathbf{a} = \frac{1}{|\mathbf{b}|}\mathbf{b} \rightarrow \mathbf{a} = \mathbf{b}$

   vii) $\mathbf{a} = m\mathbf{b} \Rightarrow \mathbf{a}$ and $\mathbf{b}$ are parallel

   viii) $r(\mathbf{a}+\mathbf{b}) = r\mathbf{a} + r\mathbf{b}$

   ix) $(r+s)\,\mathbf{a} = r\mathbf{a} + s\mathbf{a}$

   x) $\mathbf{AB} = \mathbf{DC} \Rightarrow \mathbf{BC} = \mathbf{AD}$

2. 

ABCD is a parallelogram.

F is the mid-point of DC.

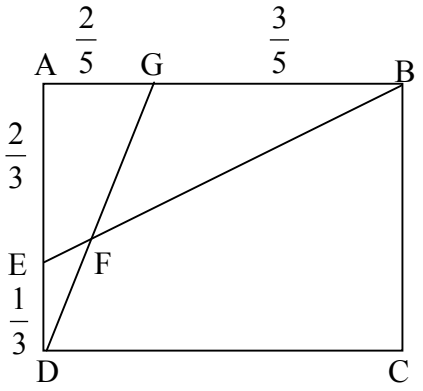
Let $\mathbf{AD} = \mathbf{a}$ and $\mathbf{AB} = \mathbf{b}$

Express the following in terms of $\mathbf{a}$ and $\mathbf{b}$

i) $\mathbf{AC}$ ii) $\mathbf{BD}$ iii) $\mathbf{DB}$ iv) $\mathbf{ED}$ v) $\mathbf{FD}$ vi) $\mathbf{FE}$

3. $\frac{1}{3}\mathbf{OP} = m\,\mathbf{OR} + \frac{5}{6}\,\mathbf{OQ}$. Find m so that P, Q, R, are collinear. Which point is between the other two and into what ratio does it divide the line segment?

4.



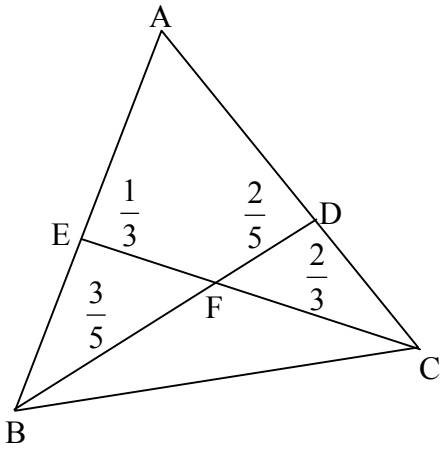
ABCD is a rectangle.

G divides AB in ratio $\frac{2}{5} : \frac{3}{5}$

E divides AD in ratio $\frac{2}{3} : \frac{1}{3}$

Find EF : FB and GF : FD

5.



In triangle ABC, D and E are on AC and AB respectively so that BD and CE intersect at F.

EF:FC = $\frac{1}{3} : \frac{2}{3}$

DF: FB = $\frac{2}{5} : \frac{3}{5}$

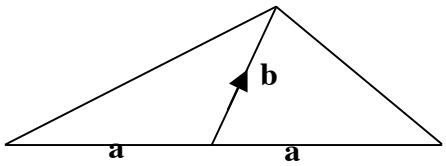
Find i) AE: EB and ii) AD:DC

6. If $\mathbf{PQ} = \frac{3}{5}\mathbf{PR} + \frac{2}{5}\mathbf{QR}$ find ratio of PQ:QR. Are P, Q, R necessarily collinear?

7. Let **a** and **b** be two vectors represented in a clockwise fashion on two adjacent sides of a regular 12-gon. Express the next consecutive clockwise side as a vector in terms of **a** and **b**.

8. Prove $|\mathbf{a} + \mathbf{b}|^2 + |\mathbf{a} - \mathbf{b}|^2 = 2(|\mathbf{a}|^2 + |\mathbf{b}|^2)$

   Hint: Use diagram



9. Show that if $(\mathbf{a} + \mathbf{b})$ is perpendicular to $(\mathbf{a} - \mathbf{b})$ then $|\mathbf{a}| = |\mathbf{b}|$

10. P, Q, R, S are centroids of faces ABC, BCD, ACD, ABD respectively of tetrahedron ABCD. Prove DP, AQ, BR, CS intersect at a point X such that DX:XP = AX:XQ = BX:XR = CX:XS = 3/4 :1/4

11. Draw a picture where | **a** | = | **b**| and | **a** + **b**| = | **a** |

12. P, Q, R, S are centroids of triangles ABC, ABD, DEF and CEF respectively. Prove PQRS is a parallelogram.

    (Hint: $\mathbf{OP} = \frac{1}{3}\ (\mathbf{OA} + \mathbf{OB} + \mathbf{OC})$

13. Prove, by vector methods, that the median of a trapezoid is parallel to the bases and equal in length to the average of the lengths of the bases.

14. Given ABCD is a regular tetrahedron with E and F as mid-points of AD and BC respectively, prove that EF is perpendicular to BC and EF is perpendicular to AD.

15. Prove that if AM, BN and CP are medians of triangle ABC then **AM**+**BN**+**CP** =**0**.

Chapter 1 Review Exercise

1. i) True ii) True iii) False iv) False v) False vi)False
   vii) True viii) True ix) True x) True

2. i) $\mathbf{a} + \mathbf{b}$ ii) $\mathbf{a} - \mathbf{b}$ iii) $\mathbf{b} - \mathbf{a}$ iv) $\frac{1}{2}\ \mathbf{a} - \frac{1}{2}\ \mathbf{b}$ v) $-\frac{1}{2}\ \mathbf{b}$

vi) $-\frac{1}{2}\mathbf{a}$ 3. m = -1/2. Q is between R and P. 2/5: 3/5

4. i) $\frac{2}{11} : \frac{9}{11}$ and ii) $\frac{6}{11} : \frac{5}{11}$ 5. i) $\frac{3}{5} : \frac{2}{5}$ ii) $\frac{5}{9} : \frac{4}{9}$

6. $\frac{5}{7} : \frac{2}{7}$ Yes. 8. $\sqrt{3}\ \mathbf{b} - \mathbf{a}$

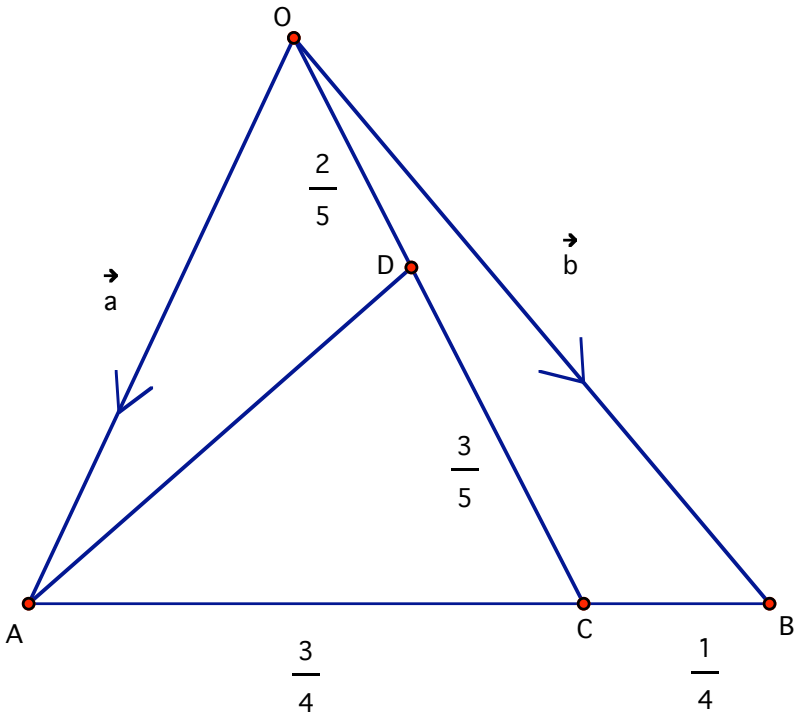
PRACTICE TEST 1 on Chapter 1

1. True or False?
   a) **OA** = **OB** + **BC** + **CA** for all points O, A, B, C
   b) (**OA** = **OB** + **BC** + **CA)** implies A, B and C are collinear
   c) $\left|\vec{a}+\vec{b}\right| \geq |\vec{a}|$
   d) If $\vec{a}=\vec{b}$ and $|\vec{c}|=\left|\vec{d}\right|$ then $|\vec{a}+\vec{c}|=\left|\vec{b}+\vec{d}\right|$
   e) Any three points are coplanar?
   f) $|\vec{a}|+\left|\vec{b}+\vec{c}\right|=|\vec{c}|$
2. Draw a picture illustrating the FALSITY of the implication

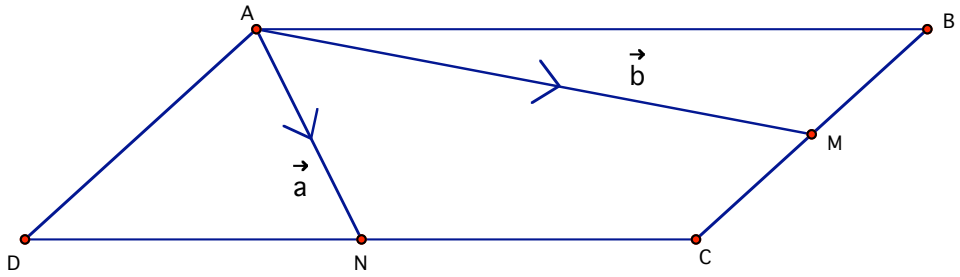
$$\left|\vec{a}+\vec{b}\right| \geq |\vec{a}| \text{ implies } \left|\vec{a}-\vec{b}\right| \leq |\vec{a}|$$

3. In the diagram below $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$, AC : CD = $\frac{3}{4}:\frac{1}{4}$ and OD : DC = $\frac{2}{5}:\frac{3}{5}$.

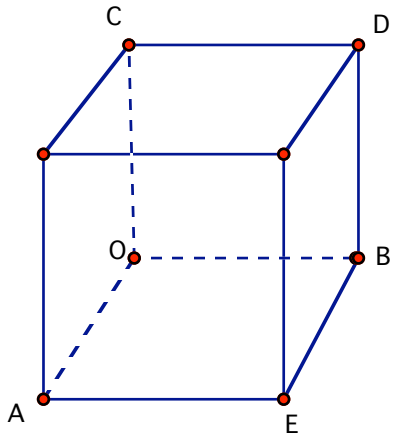
   Express $\overrightarrow{AD}$ in terms of $\vec{a}$ and $\vec{b}$.



4. In the diagram below ABCD is a parallelogram. M and N are midpoints of BC and DC respectively. Let $\overrightarrow{AN} = \vec{a}$ and let $\overrightarrow{AM} = \vec{b}$. Express a) $\overrightarrow{AB}$ in terms of $\vec{a}$ and $\vec{b}$ (b) $\overrightarrow{AD}$ in terms of $\vec{a}$ and $\vec{b}$.



5. The figure shown is a cube where $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = \vec{c}$.
   a) Let N be the midpoint of AD (not drawn). Express $\overrightarrow{ON}$ in terms of $\vec{a}, \vec{b}, \vec{c}$.
   b) Let M be the midpoint of EC. Express $\overrightarrow{OM}$ in terms of $\vec{a}, \vec{b}, \vec{c}$.
   c) Do M and N *have to* be the same point?



Practice Test 1 on Chapter 1 Answers

1.a) T b) F c) F d) F e) T f) F

2. ----- 3. $-\frac{9}{10}\vec{a} + \frac{3}{10}\vec{b}$ 4. a) $\overrightarrow{AB} = \frac{4}{3}\vec{b} - \frac{2}{3}\vec{a}$ (b) $\overrightarrow{AD} = \frac{4}{3}\vec{a} - \frac{2}{3}\vec{b}$

5. a) $\frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} + \frac{1}{2}\vec{c}$ (b) $\frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} + \frac{1}{2}\vec{c}$ (c) Yes

Practice Test 2 on Chapter 1

1. True or false? No explanation required.

   a) $|\vec{a}+\vec{b}| \geq |\vec{a}|$

   b) $|\vec{a}+\vec{b}| = |\vec{a}+\vec{c}| \Rightarrow |\vec{b}| = |\vec{c}|$

   c) $\vec{a}+\vec{b} = \vec{a}+\vec{c} \Rightarrow \vec{b} = \vec{c}$

   d) Any three points are coplanar

   e) Any three vectors are coplanar

   f) $\overrightarrow{AB} = \overrightarrow{AC} + 2\overrightarrow{CD} \Rightarrow$ A, B, C and D are coplanar

   g) $\overrightarrow{RF} = \overrightarrow{SW} \Rightarrow \overrightarrow{RS} = \overrightarrow{FW}$

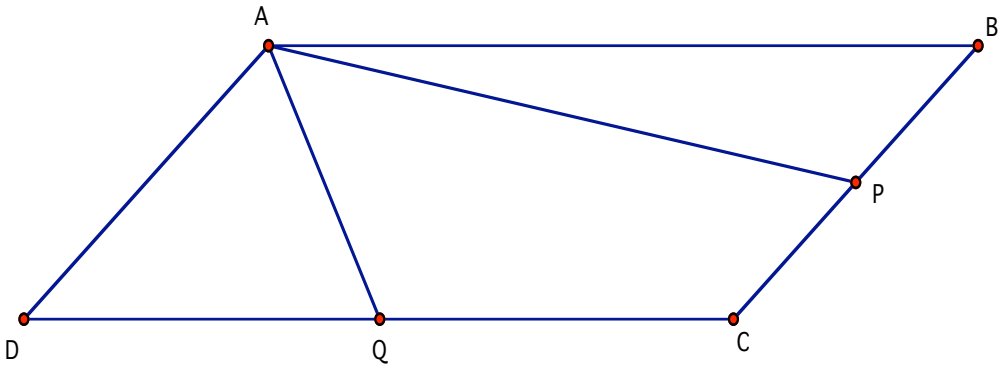
   h) $m\vec{a} + n\vec{a} = (m+n)\vec{a}$

   i) If $|\vec{a}| = |\vec{b}|$ and $|\vec{c}| = |\vec{d}|$ then $|\vec{a}+\vec{c}| = |\vec{b}+\vec{d}|$

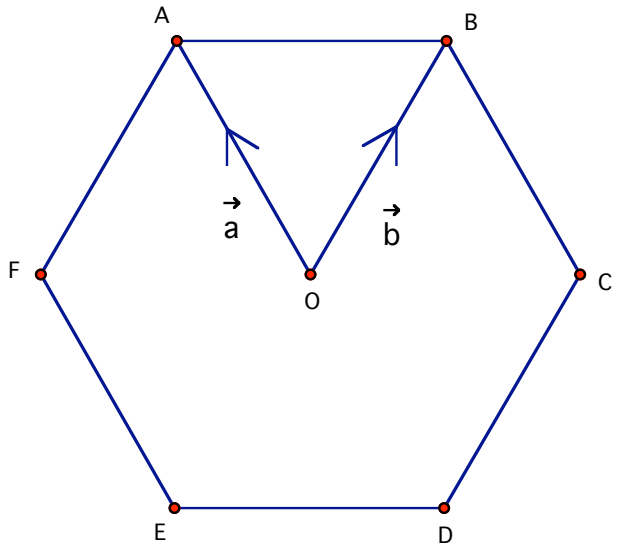
   j) $\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{DB}$ implies D and C are the same point

2. Draw a large, clear, neat diagram in which $|\vec{a}| = 2|\vec{b}|$ and $|\vec{a}| = |\vec{a}+\vec{b}|$. You need not explain, but please show the directions of the vectors.

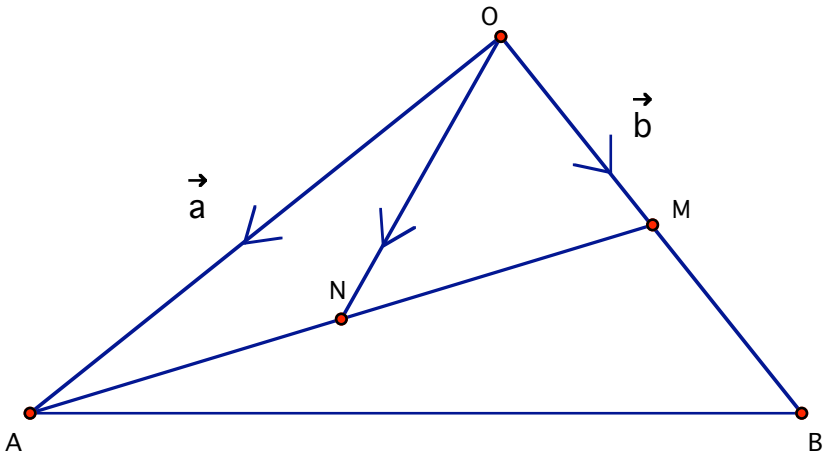
3. In the diagram below, ABCD is a parallelogram, P is the mid-point of BC and Q is the mid-point of DC. Express $\overrightarrow{BD}$ in terms of $\overrightarrow{AP}$ and $\overrightarrow{AQ}$ only.



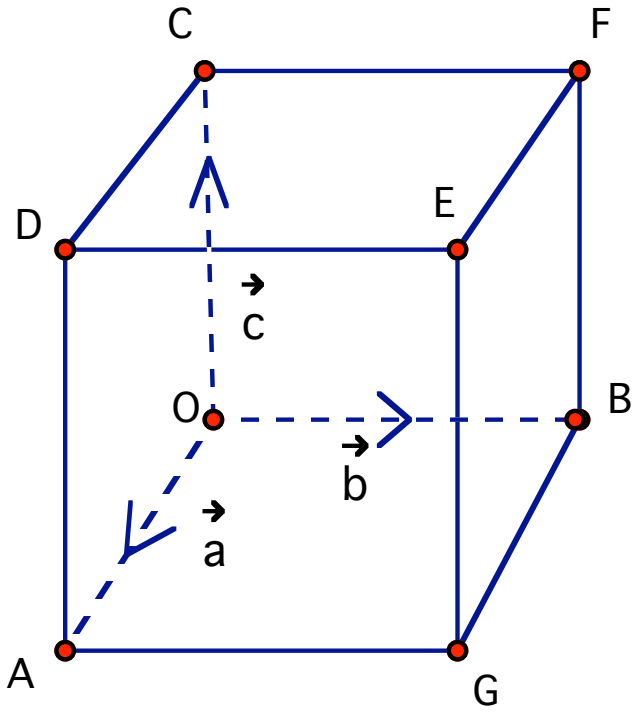
4. ABCDEF is a regular hexagon and O is the centre of the hexagon. Let $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$. Express $\overrightarrow{BF}$ in terms of $\vec{a}$ and $\vec{b}$.



5. In the diagram below, let $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$. M is the mid-point of OB and N is the mid-point of AM.
   a. Express $\overrightarrow{ON}$ in terms of $\vec{a}$ and $\vec{b}$.
   b. (Bonus) If ON is extended to meet AB at point H, in what ratio would H divide AB? That is, determine AH : HB. What is ON : NH?



6. OADCFEGB is a cube (all sides equal). Let $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$ and let $\overrightarrow{OC} = \vec{c}$. Let H be the mid-point of GC (not shown).
   a. Express $\overrightarrow{HC}$ in terms of $\vec{a}, \vec{b}, \vec{c}$.
   b. (Bonus) Find $\angle OHA$ (you will need a calculator)



Practice Test 2 on Chapter 1 Answers

1. a) F b) F c) T d) T e) F f) T g) T h) T i) F j) T k) F

2. Answers may vary.

3. $2\overrightarrow{AQ} - 2\overrightarrow{AP}$

4. $\vec{a} - 2\vec{b}$

5. a) $\frac{1}{2}\vec{a} + \frac{1}{4}\vec{b}$

6. a) $-\frac{1}{2}\vec{a} - \frac{1}{2}\vec{b} + \frac{1}{2}\vec{c}$ b) 70.52°

Practice Test 3 on Chapter 1

1. True or False (no explanation required)

   a) $\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OB}$

   b) $\left|\overrightarrow{OA}\right|+\left|\overrightarrow{AB}\right|=\left|\overrightarrow{OB}\right|$

   c) $\overrightarrow{AB}=m\overrightarrow{AC}$ implies that A, B and C are collinear

   d) $\overrightarrow{OA}+\overrightarrow{OB}=\overrightarrow{OC}$ implies that O, A, B, C are coplanar

   e) $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=\overrightarrow{OD}$ implies that O, A, B, C, D are coplanar

   f) $\left(|\vec{a}|\geq\left|\vec{b}\right| \quad and \quad \left|\vec{b}\right|\geq|\vec{c}|\right)$ together implies that $\left|\vec{a}+\vec{b}\right|\geq|\vec{a}+\vec{c}|$

   g) $\left(\vec{a}+\vec{b}\right)+\vec{c}=\vec{a}+\left(\vec{b}+\vec{c}\right)$

   h) $\vec{a}+2\vec{b}=\vec{c}$ implies that $\vec{a},\vec{b},\vec{c}$ are parallel vectors

   i) $\vec{a}+2\vec{b}=\vec{c}$ implies that $|\vec{a}|+\left|2\vec{b}\right|=|\vec{c}|$

   j) $\vec{a}+2\vec{b}=\vec{c}$ implies that $\vec{a},\vec{b},\vec{c}$ are coplanar

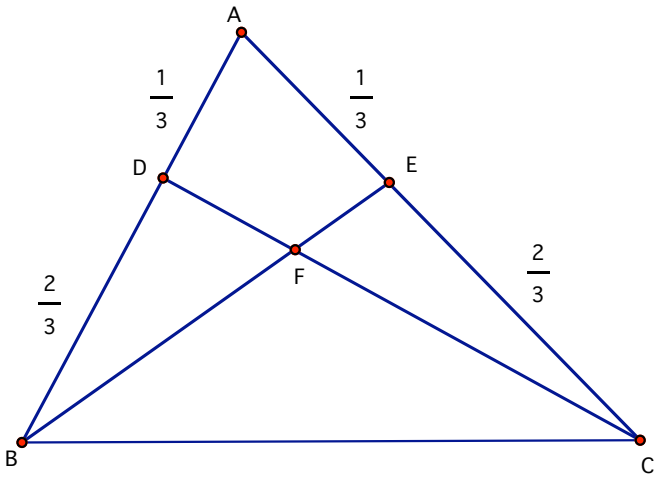
   k) $5\overrightarrow{OP}=11\overrightarrow{OQ}-6\overrightarrow{OR}$ implies P, Q, R are collinear

   l) $|\vec{a}|+\left|\vec{b}\right|\geq\left|\vec{a}+\vec{b}\right|$

   m) $|\vec{a}|+\left|\vec{b}\right|+|\vec{c}|\geq\left|\vec{a}+\vec{b}+\vec{c}\right|$

   n) If $|\vec{a}|=4$ *and* $\left|\vec{b}\right|=7$ then it is possible for $\left|\vec{a}-\vec{b}\right|=2$.

2. In the diagram below AE : EC = $\frac{1}{3} : \frac{2}{3}$ and AD : DB = $\frac{1}{3} : \frac{2}{3}$.

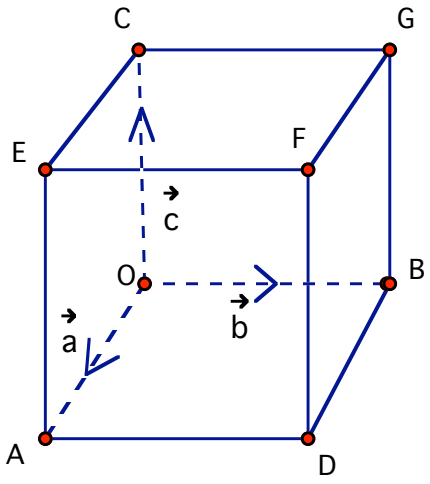


a) Find the ratio BF : FE

b) If AF were extended to point M on BC would M necessarily be the midpoint of BC? A little explanation please.

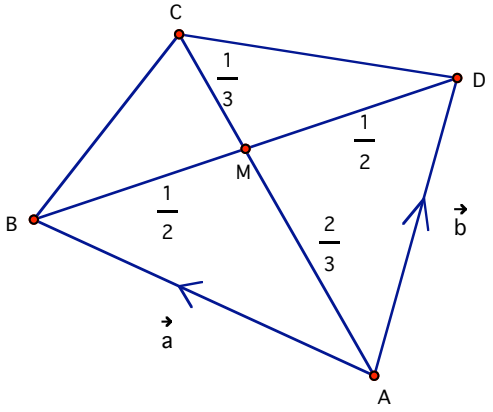
c) Find the location of point N on BC so that $\overrightarrow{FN}$ is parallel to $\overrightarrow{AB}$.

3. In the diagram below AOCEFGBD is a cube. P is the mid-point of GB. Let $\overrightarrow{OA} = \vec{a}$ and let $\overrightarrow{OB} = \vec{b}$ and let $\overrightarrow{OC} = \vec{c}$. Express $\overrightarrow{AD}$ in terms of $\vec{a}, \vec{b}, \vec{c}$.



4. Draw a NEAT, LARGE, CLEAR diagram in which $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}+\vec{c}\right|$ and $\left|\vec{b}+\vec{c}\right|=\left|\vec{a}\right|$. An approximate figure will do.

5. In the diagram below, ABCD is a quadrilateral in which AC intersects BD at M so that M is the mid-point of BD and such that AM : MC = $\frac{2}{3}:\frac{1}{3}$. Express $\overrightarrow{BD}$ in terms of $\vec{a}$ and $\vec{b}$.



6. G is the centroid of triangle ABC. Let $\overrightarrow{AB}=\vec{a}$ and let $\overrightarrow{AC}=\vec{b}$. Express $\overrightarrow{GA}+\overrightarrow{GB}$ in terms of $\vec{a}$ and $\vec{b}$.

Practice Test 3 on Chapter 1 Answers

1. a) T b) F c) T d) T e) F f) F g) T h) F i) F j) T k) T
   l) T m) T n) T

2. a) $\frac{3}{4}:\frac{1}{4}$

   b) Yes

   c) BN : NC = $\frac{1}{4}:\frac{3}{4}$

3. $-2\vec{a}+3\vec{b}+\vec{c}$ 4. ------

5. $\overrightarrow{BC}=-\frac{1}{4}\vec{a}+\frac{3}{4}\vec{b}$ 6. $\frac{1}{3}\vec{a}-\frac{2}{3}\vec{b}$